# ON THE SCIENTIFIC JUSTIFICATION OF A CONCEPTUAL NOTATION

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Time and again, in the more abstract regions of science, the lack of a means of avoiding misunderstandings on the part of others, and also errors in one's own thought, makes itself felt. Both shortcomings have their origin in the imperfection of language, for we do have to use sensible symbols to think.

Our attention is directed by nature to the outside. The vivacity of sense-impressions surpasses that of memory-images to such an extent that, at first, sense-impressions determine almost by themselves the course of our ideas, as is the case in animals. And we would scarcely ever be able to escape this dependency if the outer world were not to some extent dependent upon us.

Even most animals, through their ability to move about, have an influence on their sense-impressions: they can flee some, seek others. And they can even effect changes in things. Now man has this ability to a much greater degree; but nevertheless, the course of our ideas would still not gain its full freedom from this ability alone: it would still be limited to that which our hand can fashion, our voice intone, without the great invention of symbols which call to mind that which is absent, invisible, perhaps even beyond the senses.

I do not deny that even without symbols the perception of a thing can gather about itself a group of memory-images; but we could not pursue these further: a new perception would let these images sink into darkness and allow others to emerge. But if we produce the symbol of an idea which a perception has called to mind, we create in this way a firm, new focus about which ideas gather. We then select another from these in order to elicit its symbol. Thus we penetrate step by step into the inner world of our ideas and move about there at will, using the realm of sensibles itself to free ourselves from its constraint. Symbols have the same importance for thought that discovering how to use the wind to sail against the wind had for navigation.

Thus, let no one despise symbols! A great deal depends upon choosing them properly. And their value is not diminished by the fact that, after long practice, we need no longer produce external symbols, we need no longer speak out loud in order to think; for we think in words nevertheless, and if not in words, then in mathematical or other symbols.

Also, without symbols we would scarcely lift ourselves to conceptual thinking. Thus, in applying the same symbol to different but similar things, we actually no longer symbolize the individual thing, but rather what the similars have in common: the concept. This concept is first gained by symbolizing it; for since it is, in itself, imperceptible, it requires a perceptible representative in order to appear to us.

This does not exhaust the merits of symbols; but it may suffice to demonstrate their indispensability. Language proves to be deficient, however, when it comes to protecting thought from error. It does not even meet the first requirement which we must place upon it in this respect; namely, being unambiguous. The most dangerous cases of ambiguity are those in which the meanings of a word are only slightly different, the subtle and yet not unimportant variations. Of the many examples of this kind of ambiguity only one frequently recurring phenomenon may be mentioned here: the same word may serve to designate a concept and a single object which falls under that concept. Generally, no strong distinction is made between concept and individual. "The horse" can denote a single creature; it can also denote the species, as in the sentence: "The horse is an herbivorous animal." Finally, horse can denote a concept, as in the sentence: "This is a horse."

Language is not governed by logical laws in such a way that mere adherence to grammar would guarantee the formal correctness of thought processes. The forms in which inference is expressed are so varied, so loose and vague, that presuppositions can easily slip in unnoticed and then be overlooked when the necessary conditions for the conclusion are enumerated. In this way, the conclusion obtains a greater generality than it justifiably deserves.

Even such a conscientious and rigorous writer as Euclid often makes tacit use of presuppositions which he specifies neither in his axioms and postulates nor in the premisses of the particular theorem being proved. Thus, in the proof of the nineteenth theorem of the first book of The Elements (in every triangle, the largest angle lies opposite the largest side), he tacitly uses the statements:

(1) If a line segment is not larger than a second one, the former is equal to or smaller than the latter.

(2) If an angle is the same size as a second one, the former is not larger than the latter.

(3) If an angle is smaller than a second one, the former is not larger than the latter.

Only by paying particular attention, however, can the reader become aware of the omission of these sentences, especially since they seem so close to being as fundamental as the laws of thought that they are used just like those laws themselves.

A strictly defined group of modes of inference is simply not present in ordinary language, so that on the basis of linguistic form we cannot distinguish between a "gapless" advance in the argument and an omission of connecting links. We can even say that the former almost never occurs in ordinary language, that it runs against the feel of language because it would involve an insufferable prolixity. In ordinary language, logical relations are almost always only hinted at—left to guessing, not actually expressed.

The only advantage that the written word has over the spoken word is permanence: with the written word, we can review a train of thought many times without fear that it will change; and thus we can test its validity more thoroughly. In this process of testing, since insufficient security lies in the nature of the word-language itself, the laws of logic are applied externally like a plumb-line. But even so, mistakes easily escape the eye of the examiner, especially those which arise from subtle differences in the meanings of a word. That we nevertheless find our way about reasonably well in life as well as in science we owe to the manifold ways of checking that we have at our disposal. Experience and space perception protect us from many errors. Logical rules, externally applied, furnish little protection, as is shown by examples from disciplines in which the means of checking begin to fail. These rules have failed to defend even great philosophers from mistakes, and have helped just as little in keeping higher mathematics free from error, because they have always remained external to content.

The shortcomings stressed here are rooted in a certain softness and instability of ordinary language, which nevertheless is necessary for its versatility and potential for development. In this respect, ordinary language can be compared to the hand, which despite its adaptability to the most diverse tasks is still inadequate. We build for ourselves artificial hands, tools for particular purposes, which work with more accuracy than the hand can provide. And how is this accuracy possible? Through the very stiffness and inflexibility of parts the lack of which makes the hand so dextrous. Word-language is inadequate in a similar way. We need a system of symbols from which every ambiguity is banned, which has a strict logical form from which the content cannot escape.

We may now ask which is preferable, audible symbols or visible ones. The former have, first of all, the advantage that their production is more independent of external circumstances. Furthermore, much can be made in particular of the close kinship of sounds to inner processes. Even their form of appearance, the temporal sequence, is the same; both are equally fleeting. In particular, sounds have a more intimate relation to the emotions than shapes and colours do; and the human voice with its boundless flexibility is able to do justice to even the most delicate combinations and variations of feelings. But no matter how valuable these advantages may be for other purposes, they have no importance for the rigour of logical deductions. Perhaps this intimate adaptability of audible symbols to the physical and mental conditions of reason has just the disadvantage of keeping reason more dependent upon these.

It is completely different with visible things, especially shapes. They are generally sharply defined and clearly distinguished. This definiteness of written symbols will tend to make what is signified also more sharply defined; and just such an effect upon ideas must be asked for the rigour of deduction. This can be achieved, however, only if the symbol directly denotes the thing symbolized.

A further advantage of the written symbol is greater permanence and immutability. In this way, it is also similar to the concept—as it should be—and thus, of course, the more dissimilar to the restless flow of our actual thought processes. Written symbols offer the possibility of keeping many things in mind at the same time; and even if, at each moment, we can only concentrate upon a small part of these, we still retain a general impression of what remains, and this is immediately at our disposal whenever we need it.

The spatial relations of written symbols on a two-dimensional writing surface can be employed in far more diverse ways to express inner relationships than the mere following and preceding in one-dimensional time, and this facilitates the apprehension of that to which we wish to direct our attention. In fact, simple sequential ordering in no way corresponds to the diversity of logical relations through which thoughts are interconnected.

Thus, the very properties which set the written symbol further apart than the spoken word from the course of our ideas are most suited to remedy certain shortcomings of our make-up. Therefore, when it is not a question of representing natural thought as it actually took shape in reciprocal action with the word-language, but concerns instead the supplementation of the onesidedness of thinking which results from a close connection with the sense of hearing, then the written symbol will be preferable. Such a notation must be completely different from all word-languages in order to exploit the peculiar advantages of written symbols. It need hardly be mentioned that these advantages scarcely come into play at all in the written word. The relative position of the words with respect to each other on the writing surface depends to a large extent upon the length of the lines of print and is, thus, without importance.

There are, however, completely different kinds of notation which better exploit these advantages. The arithmetic language of formulas is a conceptual notation since it directly expresses the facts without the intervention of speech. As such, it attains a brevity which allows it to accommodate the content of a simple judgement in one line. Such contents—here equations or inequalities—as they follow from one another are written under one another. If a third follows from two others, we separate the third from the first two with a horizontal stroke, which can be read "therefore." In this way, the two-dimensionality of the writing surface is utilized for the sake of perspicuity. Here the deduction is stereotyped, being almost always based upon identical transformations of identical numbers yielding identical results. Of course, this is by no means the only method of inference in arithmetic; but where the logical progression is different, it is generally necessary to express it in words. Thus, the arithmetic language of formulas lacks expressions for logical connections; and, therefore, it does not merit the name of conceptual notation in the full sense.

Exactly the opposite holds for the symbolism for logical relations originating with Leibniz and revived in modern times by Boole, R. Grassmann, S. Jevons, E. Schröder, and others. Here we do have the logical forms, though not entirely complete; but content is lacking. In these cases, any attempt to replace the single letters with expressions of contents, such as analytic equations, would demonstrate with the resulting imperspicuity, clumsiness—even ambiguity—of the formulas how little suited this kind of symbolism is for the construction of a true conceptual notation.

I would demand the following from a true conceptual notation: It must have simple modes of expression for the logical relations which, limited to the necessary, can be easily and surely mastered. These forms must be suitable for combining most intimately with a content. Also, such brevity must be sought that the two-dimensionality of the writing surface can be exploited for the sake of perspicuity. The symbols for denoting content are less essential. They can be easily created as required, once the general logical forms are available. If the analysis of a concept into its ultimate components does not succeed or appears unnecessary, we can be content with temporary symbols.

It would be easy to worry unnecessarily about the feasibility of the matter. It is impossible, someone might say, to advance science with a conceptual notation, for the invention of the latter already presupposes the completion of the former. Exactly the same apparent difficulty arises for ordinary language. This is supposed to have made reason possible, but how could man have invented language without reason? Research into the laws of nature employs physical instruments; but these can be produced only by means of an advanced technology, which again is based upon knowledge of the laws of nature. The apparently vicious circle is resolved in each case in the same way: an advance in physics results in an advance in technology, and this makes possible the construction of new instruments by means of which physics is in turn advanced. The application of this example to our case is obvious.

Now I have attempted to supplement the formula language of arithmetic with symbols for the logical relations in order to produce—at first just for arithmetic—a conceptual notation of the kind I have presented as desirable. This does not rule out the application of my symbols to other fields. The logical relations occur everywhere, and the symbols for particular contents can be so chosen that they fit the framework of the conceptual notation. Be that as it may, a perspicuous representation of the forms of thought has, in any case, significance extending beyond mathematics. May philosophers, then, give some attention to the matter!