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CHAPTER 23

Short-Range Forces

ROBERT FOX

NEARLY ALL of Laplace's work in physics from 1802 was characterized by an interest in what he saw as the outstanding problems of the Newtonian tradition; in this respect his attempt to correct Newton's expression for the velocity of sound was typical. From 1805, however, his interest in Newtonian problems assumed a more mathematical character. As we shall see, much of the work to which he now turned was eventually to be severely criticized. But, whatever its shortcomings—and it did little to enhance Laplace's reputation in his later life—it contained results of enduring value, most notably in the theory of capillary action. No less importantly, it served to tighten the bond between mathematics and physics. This is not to imply that Laplace was in any sense the founder of mathematical physics: there were too many precursors in the eighteenth century for that claim to be sustained. But he did make a major contribution to the mathematization of a discipline that had hitherto been predominantly experimental.

The increasingly mathematical thrust of Laplace's work in physics is very apparent in the studies of molecular physics that he pursued, and encouraged others to pursue, for the rest of his life. But the interest was not a new one. As early as 1783, when he composed his *Théorie du mouvement et de la figure elliptique des planètes*, he elaborated a suggestion he had already advanced with Lavoisier in the *Mémoire sur la chaleur*.[1] This expressed his belief that optical refraction, capillary action, the cohesion of solids, their crystalline properties, and even chemical reactions were the results of an attractive molecular force, gravitational in nature and even identical with gravity.[2] Almost twenty years earlier, near the beginning of his career, he had remarked in the dual probability-gravitation memoir that analogy gives us every reason to suppose that gravity operates between all the particles of matter, extending down to the shortest ranges.[3] He repeated and elaborated the

[1] [1783a].

[2] [1784a], xii–xiii; see chapter 16. On the belief as a central theme of Laplacian physics and on the rise and decline of this style of physics, see Fox (1974).

[3] [1776a, 2°]; see chapter 5.

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speculation in the first edition of *Exposition du système du monde*,[4] looking forward to the day when the law governing the force would be understood and when "we shall be able to raise the physics of terrestrial bodies to the state of perfection to which celestial physics has been brought by the discovery of universal gravitation." In this comment, which is reminiscent of the speculations on molecular forces in the Queries of Newton's *Opticks*, there lay the nucleus of a program that guided Laplace's own research in physics and that of several distinguished pupils until the 1820s.

Despite the early adumbrations of the program, it was not until 1805 that Laplace began publishing on the individual problems it raised. By then, interest in molecular forces treated in the Newtonian manner had been greatly stimulated in France by Berthollet's work on chemical affinity, in particular by his *Essai de statique chimique* (1803), in which chemical reactions were explained in terms of short-range attractive forces, supposedly of a gravitational nature, of precisely the kind postulated by Laplace in his physics. It seems likely that Laplace was strongly influenced both by the *Essai* and by Berthollet himself, whose close friend he had been since the 1780s, and that the influence was reciprocal. In any event, Laplace's first work in molecular physics was published just two years after the publication of the *Essai*, in Book X of the fourth volume of *Traité de mécanique céleste* (1805)[5] and in two supplements to the book published in 1806 and 1807.[6]

It is a measure of Laplace's closeness to the Newtonian tradition that these first studies of molecular forces were concerned with optical refraction and capillary action. Both were manifestations of action at a distance on the molecular scale that had been of special interest to eighteenth-century Newtonians such as Clairaut and Buffon as well as to Newton himself. The contributions of Clairaut appear to have been especially relevant to Laplace's work. In the 1740s, for example, Clairaut had ascribed "the roundness of drops of fluid, the elevation and depression of liquids in capillary tubes, the bending of rays of light, etc." to gravitational forces that become large at small (i.e., molecular) distances.[7] But despite the attention that both Clairaut and Buffon had paid to these and related theoretical problems, there was still no satisfactory answer by the end of the eighteenth century. In particular, although it was generally accepted that the force between the particles of ordinary matter (in the case of capillary action) and between the

[4] [1796], *OC*, **2**, pp. 196–98.

[5] See especially Chapter 1, pp. 231–76, *OC*, **4**, pp. 233–77.

[6] *OC*, **4**, pp. 349–417, 419–98.

[7] Clairaut, "Du système du monde dans les principes de la gravitation universelle," *MARS* (1745/1749), pp. 329–64 (338).

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particles of ordinary matter and the particles of light (in the case of refraction) diminishes rapidly with distance, it had proved impossible to determine the law relating force and distance. Clairaut had tried to account for the intense short-range forces by suggesting that the law of gravitational force should contain a term inversely proportional to the fourth power of the distance, $1/r^4$.[8] Buffon, by contrast, had upheld the $1/r^2$ law, although he had observed that such a law would be modified at short range by the shape of the particles of matter.[9] Recognizing the intractability of the problem, Laplace proposed a much simpler solution that could be applied in all branches of molecular physics. In treatments that made good use of mathematical techniques developed in his earlier work on celestial mechanics, he showed that the precise form of the law was unimportant and that perfectly satisfactory theories could be given by simply making the traditional assumption that the molecular forces act only over insensible distances.

The treatment of refraction in Book X of *Mécanique céleste* centered on the specific problem of atmospheric refraction, a matter of practical as well as theoretical concern to Laplace and his colleagues at the Bureau des Longitudes. The whole discussion was conducted in terms of the corpuscular theory of light, the truth of which was assumed for the purposes of the calculation (though not explicitly endorsed as a physical reality). According to Laplace, the path of a corpuscle of light passing through the successive layers of the earth's atmosphere is determined by the varying attractive forces exerted on it by the particles of air. The measure of these forces was what Laplace, following Newton, called the refracting force (*force réfringente*), a quantity equal to $(\mu^2 - 1)$, where μ is the refractive index of the air. In this analysis, $(\mu^2 - 1)$ is proportional to the increase in the square of the velocity of the incident corpuscles of light, and hence, in accordance with the normal laws of dynamics, it measures the force of attraction to which they are subject. In deriving his extremely complicated differential equation for the motion of light through the atmosphere, Laplace had to assume not only the short-range character of the forces to which light corpuscles were subjected but also that the refracting force was proportional to the density of the air, ρ. In order to integrate the equation, it was necessary to make further, speculative assumptions concerning the variation of ρ with altitude (the subject of a long and properly tentative section) and to allow for the effect on ρ of the air's humidity. The result was a method of calculating the magnitude of atmospheric refraction for

[8] Ibid., pp. 337–39.

[9] Buffon, "De la nature. Seconde vue" [1765], in his *Histoire naturelle, générale et particulière*, 36 vols. (Paris, 1749–1804), **13**, pp. xii–xv.

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which Laplace claimed complete reliability at any but small angles of elevation.

Despite important subsequent refinements, the core of Laplace's theory of capillary action, as expounded in the two supplements to Book X of *Mécanique céleste*, continued to guide discussions throughout the nineteenth century, and it even survives in modern textbooks.[10] However, its roots lie as firmly in the vain quest for a comprehensive physics of short-range forces as do those of his work on refraction. As in the theory of refraction, it was crucial that the forces exerted by the particles of matter on one another could be ignored at any but insensible distances (although it was equally important that these distances be finite—an assumption that distinguished Laplace's theory from others in which adhesion was seen as the cause of capillary phenomena). Hence Laplace was glad to invoke Hauksbee's observation that the height to which a liquid rises in a capillary tube is independent of the thickness of the walls of the tube.

In each of the two supplements on capillary action Laplace presented a quite distinct version of his theory. In the first, he arrived at a general differential equation of the surface of a liquid in a capillary tube by considering the force acting on an infinitely narrow canal of the liquid parallel to the axis of the tube. In the second, he treated the equilibrium of the column of liquid in a capillary tube by considering the forces acting upon successive cylindrical laminae of the liquid parallel to the sides of the tube. The two versions were in no sense inconsistent with each other, although in a number of applications the second version proved to be somewhat simpler and more fruitful.

Laplace was concerned, above all, to demonstrate the close agreement between his theory and experiment, and much of both supplements was devoted to this task. One of his most striking successes, in the first supplement, was confirmation of a proof, obtained by solving the differential equation of the liquid surface that the elevation of a liquid in a capillary tube is very nearly in inverse proportion to the tube's diameter.[11] In the same supplement and using the same version of the theory, he also showed that the insertion of a tube of radius r_1 along the axis of a hollow tube of slightly larger radius r_2 causes the liquid between the tubes to rise to a height equal to that to which it would rise in a circular capillary tube of radius $(r_2 - r_1)$;[12] in this way,

[10] See, for example, Champion and Davy (1952), pp. 172–77, and the brief study of Laplace's theory in Bikerman (1975). For a more recent account that places Laplace's investigation of both capillarity and refraction in the tradition of work pursued by Clairaut, see Heilbron (1993), pp. 150–65.

[11] (86), 23–25, *OC*, **4**, pp. 372–74.

[12] (86), 25–28, *OC*, **4**, pp. 374–78.

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he confirmed a well-known observation made but not explained by Newton. Among the other classic problems treated in the supplements were the behavior of a drop of liquid in capillary tubes of various shapes (including conical tubes), the rise of liquids between parallel or nearly parallel plates, the shape of a drop of mercury resting on a flat surface, and the force drawing together parallel plates separated by a thin film of liquid.

The importance, for Laplace's theory, of the short-range character of the molecular forces cannot be overstated. The assumption allowed him repeatedly to set aside small terms involving the square of the distance, and, with the aid of this simplification, to pursue mathematical investigations that would otherwise have been impossible. It was in accordance with his belief that capillarity is a consequence of intermolecular action at a distance (albeit at a very small distance) that he was able to calculate the relative magnitude of the attractive force between the particles composing the liquid (F_1) and the force between the particles of the liquid and those of the tube (F_2) and to define the conditions that determined the shape of the liquid's surface. Neglecting variations in density near the surface of the liquid and the walls of the tube, he showed that if $F_2 > F_1/2$, the surface must be concave; otherwise it must be convex, being, in the limiting case of $F_2 = 0$, a convex hemisphere.[13]

Even as the supplements on capillary action were being written, the comparison between theory and observation was being carried still further in experiments, performed at Laplace's request, by Gay-Lussac, Haüy, and the engineer Jean-Louis Trémery.[14] These experiments gave the theory added plausibility, as Laplace himself was always ready to observe; and they certainly helped it to survive the criticism of his most important contemporary rival in the treatment of capillarity, Thomas Young. By comparison, Young's theory, which was based on the concept of surface tension rather than intermolecular attraction, was obscure and unmathematical.[15] Yet Laplace's theory (which Young saw as "unnecessarily intricate") was not without fault, and its author led the way in making modifications. In a paper that he read to the Academy of Science in September 1819, he refined the theory to take account of the effect of heat in reducing the attractive force between the particles of a liquid;[16] the net attractive force was now taken as the difference between the innate attraction (the only force considered in the

[13] (86), 44–50, *OC*, **4**, pp. 394–401.

[14] [1806a], *OC*, **4**, pp. 403–5.

[15] Young, "An Essay on the Cohesion of Fluids," *Philosophical Transactions of the Royal Society* (1805), pp. 65–87.

[16] [1819h].

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supplements to *Mécanique céleste*) and a repulsive force that was supposed to be caused by the presence of heat. An even more important modification was made in 1831, when, in his *Nouvelle théorie de l'action capillaire*, Poisson remedied one of the most obvious weaknesses in Laplace's theory by taking account of the variations in density near the surfaces of the liquid and the material of the capillary tube.[17]

[17] Poisson, *Nouvelle théorie de l'action capillaire* (Paris, 1831); see esp. pp. 1–8.

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