CHAPTER 9

'Normal Connections' and the Law of Causality

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9.1 Introduction

The 6.3s occupy a special place in the *Tractatus*. Proposition 6 itself is the culmination of much of the earlier discussion of the book: the formal introduction of the general propositional form, encapsulating the idea that every proposition with sense is a truth-function of elementary propositions. The ensuing subsections of the 6s concern various domains of discourse – logic (6.1s), mathematics (6.2s), natural science (6.3s), and ethics (6.4s) – leading up to Wittgenstein's remarks about the method of philosophy itself in the 6.5s. The propositions of logic and mathematics are tautologies and equations, respectively, neither of which are propositions with sense (6.11, 6.21). Propositions of ethics, for their part, are impossible, for ethics 'cannot be put into words' (6.42). The discussion of natural science in the 6.3s is a distinctive section, then, for it is here that we are considering the phenomenon of senseful discourse. Indeed, in the penultimate remarks of the *Tractatus*, we are told that the correct method in philosophy 'would really be the following: to say nothing except what can be said, i.e. propositions of natural science' (6.53).

In this chapter, I want to orient the discussion around a single, obscure remark in the 6.3s, a remark that has been almost entirely neglected in the literature.[1] The remark in question is 6.361, and for reasons that will shortly become apparent it will be useful to start with the German:

> 6.361 In der Ausdrucksweise Hertz's könnte man sagen: Nur *gesetzmäßige* Zusammenhänge sind *denkbar*.[2]

[1] Hyder (2002: §6.2.3) is the only existing commentary that I am aware of.

[2] The emphasis is Wittgenstein's. For the German text, see Wittgenstein (1989). In this chapter, I use the translation by Pears and McGuinness (Wittgenstein 1974b) unless otherwise noted, and I use the standard convention for referencing remarks in the *Tractatus* by citing the line number. I also introduce a similar convention for referencing passages from Hertz's *Principles of Mechanics* by citing the section number.

When Ogden and Ramsey first translated this into English, they rendered it as follows: 'In the terminology of Hertz we might say: Only *uniform* connections are *thinkable*'.[3] However, when commenting on the draft of this translation, Wittgenstein wrote:

> 'Only uniform connexions are thinkable'. 'uniform', I think, is wrong. To get the right expression, please look up the English translation of Hertz's 'Principles of Mechanics'. In the German text it is '*gesetzmäßige Zusammenhänge*'. (Wittgenstein 1973: 35)

Turning to *Principles of Mechanics* (Hertz 1894) then (and beginning once more with the German), we find that '*gesetzmäßige Zusammenhänge*' are defined at §119: 'Ein Zusammenhang eines Systems heißt ein gesetzmäßiger, wenn er unabhängig von der Zeit besteht'. The English translation of *Principles* (Hertz 1899) renders this: 'A connection of a system is said to be normal (*gesetzmäßiger*) when it exists independently of the time'. Thus, following Wittgenstein's suggestion (as Ogden evidently failed to do), 6.361 can be translated as follows:

> 6.361 In the terminology of Hertz one might say: Only *normal* connections are *thinkable*.

It is perhaps unsurprising that translators of the *Tractatus* were not keen to rely on the translation of *Principles*. Indeed, D. H. Jones, the translator of Hertz's text, was evidently aware of the peculiarity of his choice when he included the German term in parentheses immediately after 'normal'.[4] Although no translation is perfect, one of my concrete goals in this chapter will be to argue that Wittgenstein had good reason to appeal to the English translation of *Principles*. In particular, the use of 'normal' immediately prompts the reader to ask what role this kind of connection plays in the context of Hertz's mechanics, and makes it obvious that that question must be answered first if we are to come to understand why Wittgenstein would claim that only these types of connection are thinkable (thereby implying that *abnormal* connections are *unthinkable*).

With this in mind, our first task (in Section 9.2) will be to survey the relevant aspects of Hertz's text. *Principles* begins with a long philosophical

[3] In the translation by Pears and McGuinness, we find instead: 'One might say, using Hertz's terminology, that only connections that are *subject to law* are *thinkable*'. 'Subject to law' is a more literal way to translate '*gesetzmäßig*' into English.

[4] The word 'normal' often occurs in mathematical physics. In vector calculus, for instance, normal vectors and normalized vectors are orthogonal to a given surface and have unit magnitude, respectively. Neither of these uses correspond to Hertz's 'normal connection' however.

introduction, the most well-known feature of which is Hertz's austere 'image theory' of representation. Hertz uses this image theory to compare and evaluate three different formulations of mechanics: the traditional formulation, an energetics formulation, and Hertz's own formulation. The main body of *Principles* then takes up the task of spelling out the Hertzian formulation in full detail. There are two aspects of Hertz's work that are especially salient to Wittgenstein's remarks in the 6.3s. First, there is the notion of 'distinctness' that Hertz describes as one of the main advantages of both the energetics and Hertzian formulations over the traditional formulation. And second, there is the role played by normal connections, beginning with their role in Hertz's 'fundamental law' – the only basic empirical proposition that his formulation requires.

With this in hand, we will return (in Section 9.3) to the 6.3s in the *Tractatus*, beginning with Wittgenstein's lengthy remarks about mechanics at 6.341. Here, Wittgenstein presents a somewhat elaborate analogy involving three different nets (square, triangular, and hexagonal) that are used to describe a picture formed by black spots on a white surface. The analogy is supposed to illustrate a number of ideas: that mechanics 'imposes a unified form of description on the world' and that the mere possibility of describing the world using mechanics is uninformative (what is informative, by contrast, is the 'precise *way* it is possible to describe it by these means' (6.342)). Wittgenstein also writes that we are told something about the world 'by the fact that it can be described more simply with one system of mechanics than with another' (6.342). I will argue that a satisfactory interpretation of these remarks is only possible with reference to Hertz's discussion of the different formulations of mechanics, and in particular those different formulations' relative distinctness. Turning at last to the reference to 'normal connections' at 6.361, I will argue that this is closely connected to Wittgenstein's nearby remarks about the 'law of causality' (6.32, 6.321, 6.36, and 6.362). On Wittgenstein's view, a (fundamental) abnormal connection would lie outside the realm of 'causal descriptions', indeed outside the realm of descriptions *simpliciter*. This is the key to understanding Wittgenstein's remark that only normal connections are thinkable.

Before proceeding, it is worth keeping in mind that Hertz's influence extended throughout Wittgenstein's career. Besides being one of only a handful of figures to be explicitly cited in the *Tractatus*, Wittgenstein also referred to Hertz both times he gave a public address at Cambridge[5] and came close to using a passage from *Principles* as the motto for the

[5] See McGuinness (2002a: ix).

Philosophical Investigations.[6] It would, of course, be unsurprising if Hertz's influence was especially evident in Wittgenstein's remarks about natural science (not to mention mechanics in particular). And indeed, in lengthy remarks discussing mechanics in his *Notebooks* entry of 6 December 1914, Wittgenstein mentions Hertz twice.[7] This notebook entry went on to form the basis of 6.341–6.343 in the *Tractatus*, passages that take up more than a third of the entire 6.3 sequence. Hence a close scrutiny of Hertz's mechanics – a substantive task in its own right – may well allow for a substantialy improved understanding of many of Wittgenstein's remarks about the propositions of natural science.

9.2 Hertz's Mechanics

9.2.1 An Image Theory of Representation

Hertz begins the introduction to *Principles* as follows:

> The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. . . . In endeavoring thus to draw inferences as to the future from the past, we always adopt the following process. We form for ourselves inner simulacra [*Scheinbilder*] or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents of the things pictured. (Hertz 1899: 1)

Here, concisely stated, is Hertz's image theory. The theory is notably austere – the only fundamental requirement on an image is that its necessary consequences in thought are images of the necessary consequences in nature of the thing pictured. This can be glossed as the requirement that an image provides successful predictions of the behavior of the pictured objects. In this way, we are 'enabled to be in advance of the facts, and to decide as to present affairs in accordance with the insight so obtained' (Hertz 1899: 1). According to Hertz, this matching of necessity in thought and necessity in nature ('*denknotwendig*' and '*naturnotwendig*', respectively) is the only correspondence we can hope for between our conceptions of things, on the one hand, and the things themselves, on the other.

In order to compare and evaluate different images, Hertz specifies three criteria: permissibility (*Zulässigkeit*), correctness (*Richtigkeit*), and

[6] See Kjærgaard (2002: 126) and Janik (2000: 149). [7] See Wittgenstein (1979: 35–36).

appropriateness (*Zweckmässigkeit*). In brief, we can immediately rule out as impermissible – specifically logically impermissible (*logisch unzulässig*) – any images that 'contradict the laws of our thought'. If we have two or more permissible images of a given object available, we can then rule out as incorrect (*unrichtig*) those whose 'essential relations contradict the relations of external things', those images that are not in conformity with the fundamental requirement. But in general this may still be insufficient to settle on the best image. Hertz thus spells out the criterion of appropriateness as a further means by which to choose between multiple permissible and correct images:

> Of two images of the same object that is the more appropriate which pictures more of the essential relations of the object, – the one which we may call the more distinct. Of two images of equal distinctness the more appropriate is the one which contains, in addition to the essential characteristics, the smaller number of superfluous or empty relations, – the simpler of the two. (Hertz 1899: 2)

Here it is evident that the requirement of appropriateness splits into two sub-requirements, distinctness (*Deutlichkeit*) and simplicity (*Einfachkeit*). A perfectly distinct image would capture *all* the essential relations (*wesentliche Beziehungen*) in nature, and a perfectly simple image would capture *only* those relations. To sum up, then, the most appropriate image is the one that comes the closest to capturing all and only the essential relations among the pictured objects.

Hertz employs the image theory to compare three different formulations of mechanics. The first is the 'customary representation of mechanics', the formulation 'varying in detail but identical in essence, contained in almost all the text-books which deal with the whole of mechanics' (Hertz 1899: 4). This presentation of the theory is not dissimilar to how the subject is still taught today. It begins with four primitive notions – space, time, mass, and force – and is anchored around Newton's three laws of motion. According to Hertz, although Newton's laws 'contain the seed of future developments', they do not 'furnish any general expression for the influence of rigid spatial connections' (Hertz 1899: 5). In order to accommodate all possible influences of such connections, we need to make use of d'Alembert's principle.[8] With this in hand, we have closed 'the series of independent fundamental statements which cannot be deduced from each

[8] A statement of d'Alembert's principle (or the d'Alembert-Lagrange principle) is that the total virtual work performed by all the impressed forces acting on a system, added to the total virtual work performed by all the 'forces of inertia', sums to zero (here I follow Lanczos (1962: §4)). For a discussion of d'Alembert's original formulation of the principle, see Fraser (1985).

other'; hence, we have 'a first system of principles of mechanics, and at the same time the first general image of the natural motions of material bodies' (Hertz 1899: 5).

The second formulation of mechanics, 'of much more recent origin than the first' (Hertz 1899: 14), is the *energetics* formulation. This formulation 'likes to treat the phenomena which occur in its domain as transformations of energy into new forms, and to regard as its ultimate aim the tracing back of the phenomena to the laws of the transformation of energy' (Hertz 1899: 14). Unlike the first formulation, this second formulation has not been fully worked out. Indeed, Hertz notes that perhaps there has never been a textbook 'which from the start teaches the subject from the standpoint of energy and introduces the idea of energy before the idea of force' (Hertz 1899: 15). Nevertheless, Hertz has little doubt that such a formulation is possible, and he proceeds to sketch 'the general plan according to which such a representation of mechanics must be arranged' (Hertz 1899: 15). Where the customary formulation begins with space, time, mass, and force, the energetics formulation begins with space, time, mass, and *energy*. Energy itself occurs in two generic forms: kinetic energy, associated with bodies' velocities, and potential energy, associated with their relative positions. Instead of appealing to Newton's laws of motion and d'Alembert's principle, the energetics formulation appeals to 'one of the integral principles of ordinary mechanics which involve in their statement the idea of energy' (Hertz 1899: 16). Hertz chooses Hamilton's principle for this purpose,[9] and from this point, again, everything is deductive inference: 'All that we can further add are only mathematical deductions and certain simplifications of notation' (Hertz 1899: 16). Thus, we have a second general image of the natural motions of material bodies.

The third formulation of mechanics is Hertz's own formulation, spelled out in detail in the body of *Principles*. Hertz treats only space, time, and mass as primitive, thus deriving notions of force and energy from connections (spatial relations) among masses. In order to be able to do this, Hertz assumes that 'it is possible to conjoin with the visible masses of the universe other masses obeying the same laws' (Hertz 1899: 25–26). This is Hertz's hypothesis of 'hidden masses'.[10] With its aid, Hertz can treat the effects of both energy transformations and impressed forces as due to

[9] A statement of Hamilton's principle is that, for a given initial and final state, the integral of a system's Lagrangian is stationary under arbitrary variations of the system's configuration. See Lanczos (1962: 113).

[10] For a discussion of the role played by Hertz's hidden masses and the interpretive controversy surrounding them, see Eisenthal (2018).

connections between visible and hidden masses. As with the energetics formulation, only a single principle is needed within Hertz's formulation. Here, Hertz introduces his own 'fundamental law', asserting that 'if the connections of the system could be momentarily destroyed, its masses would become dispersed, moving in straight lines with uniform velocity; but that as this is impossible, they tend as nearly as possible to such a motion' (Hertz 1899: 28). From this point, 'we can derive all the rest of mechanics by purely deductive reasoning' (Hertz 1899: 28). This completes the third general image of the natural motions of material bodies.

9.2.2 The Most Distinct Image

As already noted, Hertz applies his three criteria of permissibility, correctness, and appropriateness to evaluate the three different formulations and compare them with one another. For present purposes, it is Hertz's evaluation of each representation's appropriateness that is of particular interest, especially the first sub-requirement of distinctness.[11] Beginning, then, with the traditional formulation of mechanics, Hertz asks:

> Is this image perfectly distinct? Does it contain all the characteristics which our present knowledge enables us to distinguish in natural motions? (Hertz 1899: 10)

Hertz's answer is a definitive 'No'. In particular, Hertz argues that the domain of acceptable forces is routinely restricted beyond what is required by Newton's laws themselves. For instance, it is standard to rule out the existence of any fundamental forces whose actions would violate conservation of energy, and to assume that 'only such forces exist as can be represented as a sum of mutual actions between infinitely small elements of matter' (Hertz 1899: 10). Other properties of forces may be more controversial:

> Whether the elementary forces can only consist of attractions and repulsions along the lines connecting the acting masses; whether their magnitude is determined only by the distance or whether it is also affected by the absolute or relative velocity; whether the latter alone comes into consideration, or the acceleration or still higher differential coefficients as well – all these properties have been sometimes presumed, at other times questioned. (Hertz 1899: 10–11)

[11] For different reasons Hertz's evaluation of each representation's *simplicity* is also important, though I postpone a discussion of this matter for another time.

Though the specifics are disputed, there is general agreement that Newton's laws, on their own, do not sufficiently fix the phenomena. Similar considerations apply to the kinds of connections accommodated by d'Alembert's principle:

> It is mathematically possible to write down any finite or differential equation between coordinates and to require that it shall be satisfied; but it is not always possible to specify a natural, physical connection corresponding to such an equation: we often feel, indeed sometimes are convinced, that such a connection is by the nature of things excluded. And yet, how are we to restrict the permissible equations of condition? (Hertz 1899: 11).

In sum, straightforward appeals to Newton's laws of motion and d'Alembert's principle fail to sufficiently hone in on the phenomena insofar as we typically need to impose further constraints. In this way, the customary formulation of mechanics fails to be perfectly distinct.

When considering the advantages of the energetics formulation, Hertz turns immediately to the question of its appropriateness, 'since it is in this respect that the improvement is most obvious' (Hertz 1899: 17). In order to derive Hamilton's principle from Newton's laws of motion within the traditional formulation, one must explicitly restrict attention to *conservative* systems – systems in which the potential energy depends only on the system's instantaneous configuration. In contrast, by treating Hamilton's principle as axiomatic within the energetics formulation, the fact that all systems are conservative becomes a conclusion rather than a premise. As a consequence of this, the forces that one derives within the energetics formulation (in particular, the forces that are derived from a potential function) come equipped with a number of properties that have to be put in by hand in the traditional formulation: These forces never violate conservation of energy, can always be represented as a sum of mutual actions between infinitesimal 'elements of matter', and only depend on relative distances rather than velocities or higher order differentials. In one fell swoop, then, the energetics formulation hones in on the phenomena in a much more satisfying way. Thus, Hertz declares, 'our second image of natural motions is decidedly more distinct: it shows more of their peculiarities than the first does' (Hertz 1899: 17).

When Hertz turns to his own formulation, he declares that, in respect of distinctness and simplicity, 'we may assign to it about the same position as to the second image; and the merits to which we drew attention in the latter are also present here' (Hertz 1899: 39). Nevertheless, Hertz argues that the third image does have slight advantages. Regarding distinctness in particular, Hertz's own formulation can accommodate a larger class of rigid connections than the energetics formulation because it can accommodate *nonholonomic* constraints (constraints that depend on the differentials of the coordinates of the system).[12] Such constraints do seem to occur in nature, such as a sphere constrained to roll without slipping on a surface.[13] In this particular respect, then, Hertz's formulation is more distinct than the energetics formulation: It captures more of the essential relations in nature.[14]

Having surveyed Hertz's discussion of the three different formulations' relative distinctness, our remaining task before returning to the *Tractatus* is to delve briefly into the main body of *Principles* and consider the notion of normal (and abnormal) connections that are introduced there.

9.2.3 Normal Connections

As already noted, Hertz captures the effects of impressed forces and energy transformations by appealing to connections between visible and hidden masses. In essence, his mechanics is a mechanics of rigidly constrained systems (think of a clockwork mechanism, for example).[15] At its most general, the existence of a connection between the points composing a system simply means that knowledge of some of the components of the displacements of the points implies something about the remaining components (§109). Hertz limits his attention to *continuous* connections (§115) calling a system with only continuous connections a 'material system' (§121). Hertz also defines *internal* (§117) and *normal* connections (§119). An internal connection is one that only affects the system's configuration (the relative positions of its constitutive points), hence does not affect its absolute position. A normal connection – of particular interest in this chapter – is one that is independent of time. With these definitions in hand, Hertz can then define a *free* system as 'a material system which is subject to no other than internal and normal connections' (§122). A free system, in other words, is one whose connections are continuous, independent of absolute position, and independent of time.

12 For a discussion of holonomous and nonholonomous constraints in Hertz's mechanics, see Lützen (2005: 192–197). For a more general discussion, see Butterfield (2004: 40).

13 For Hertz's discussion of this case, see Hertz (1899: 20–21).

14 At any rate, this is what Hertz claimed on behalf of his formulation. For a detailed discussion, see Lützen (2005: §§15.3, 20.3, and 22).

15 This should not be taken too literally; Hertz's treatment of mechanical systems is, in fact, highly abstract – see Eisenthal (2018, 2022: §3).

Hertz's fundamental law is stated formally as follows:

> **Fundamental Law**. Every free system persists in its state of rest or of uniform motion in a straightest path. (§309)

Hertz's remarkable claim is that this single law, a kind of generalized principle of inertia, is sufficient to describe all the phenomena of classical mechanics.[16] In some cases, the application of the law is relatively simple, such as with certain idealized systems which can be treated as isolated.[17] The more difficult cases, however, are systems that cannot be treated as isolated; indeed, from the perspective of Hertz's framework, one way or another those systems will not appear to be free systems. How, then, can their motions be described by the fundamental law?

Hertz's strategy here is to treat any apparently unfree system as a part of a larger free system: 'Every unfree system we conceive to be a portion of a more extended free system; from our point of view there are no unfree systems for which this assumption does not obtain' (§429).[18] Hertz divides such unfree systems into two groups – *guided systems* and *systems acted on by forces*. For present purposes, it is the guided systems that are of particular interest.[19] These are defined as follows:

> A guided motion of an unfree system is any motion which the system performs while the other masses of the complete system perform a determinate and prescribed motion. A system whose motion is guided is called a guided system. (§431)

In saying that the remaining masses (of the complete system) 'perform a determinate and prescribed motion', Hertz is saying that their coordinates can be given as functions of time. This means that the connections of a guided system will in general depend on time, hence will be 'inconsistent with the requirements of normality' (§436). The appeal to guided systems now becomes a general strategy for accommodating systems which depend on time: 'we now consider every system whose equations of condition in the ordinary language of mechanics contain the time explicitly, and which in our mode of expressions is apparently abnormal, as a guided system, i.e.

[16] For a more detailed explanation of the fundamental law, see Eisenthal (2022: 286–289).

[17] Hertz gives as examples 'rigid bodies moving in free space or perfect fluids moving in closed vessels' (§316).

[18] Regarding all unfree systems as parts of larger free systems in this way may seem surprisingly speculative. To see things otherwise requires appreciating the abstract nature of Hertz's approach. For some discussion, see Eisenthal (2022: §3).

[19] For a discussion of systems acted on by forces, see Eisenthal (2021).

as a system which with other unknown masses satisfies the conditions of normality' (§436). This is how Hertz aims to show that systems that appear to depend on time (hence appear to have abnormal connections) fall under the fundamental law: the complete systems of which they are a part are, in fact, free systems with normal connections.

Although it may be a cogent strategy to treat guided systems as parts of larger free systems, this fact is not immediately helpful in determining their motions. The connections of a guided system still contain the time explicitly, and hence the fundamental law 'is not directly applicable' (§437). However, Hertz notes that a number of principles that can be derived from the fundamental law do govern the motion of guided systems. For example, the principle of least acceleration and Hamilton's principle (§§438–440) still apply when a system's connections depend on time.

We are now ready to return to the *Tractatus* and to Wittgenstein's explicit reference to normal connections at 6.361. Before turning to that particular remark, however, we should begin with a consideration of the discussion of mechanics that takes place earlier in the 6.3s.

9.3 Mechanics in the *Tractatus*

9.3.1 The Net Analogy

Wittgenstein's most explicit discussion of mechanics is also the single longest remark in the *Tractatus*. In full, 6.341 reads as follows:

> 6.341 Newtonian mechanics, for example, imposes a unified form on the description of the world. Let us imagine a white surface with irregular black spots on it. We then say that whatever kind of picture these make, I can always approximate as closely as I wish to the description of it by covering the surface with a sufficiently fine square mesh, and then saying of every square whether it is black or white. In this way I shall have imposed a unified form on the description of the surface. The form is optional, since I could have achieved the same result by using a net with a triangular or hexagonal mesh. Possibly the use of a triangular mesh would have made the description simpler: that is to say, it might be that we could describe the surface more accurately with a coarse triangular mesh than with a fine square mesh (or conversely), and so on. The different nets correspond to different systems for describing the world. Mechanics determines one form of description of the world by saying that all propositions used in the description of the world must be obtained in a given way from a given set of propositions – the axioms of mechanics. It thus supplies the bricks for

> building the edifice of science, and it says, 'Any building that you want to erect, whatever it may be, must somehow be constructed with these bricks, and with these alone'. (Just as with the number-system we must be able to write down any number we wish, so with the system of mechanics we must be able to write down any proposition of physics that we wish.)

The analogy itself is straightforward enough. If we wanted to describe a picture formed by black spots on a white surface, we could give our description a 'unified form' by laying a square net over the surface and saying which holes are black and which are white. As we could have used a triangular or hexagonal net instead of a square one, the form (the shape of mesh) is 'optional'. However, one of these forms might prove to be simpler than another; 'it might be that we could describe the surface more accurately with a coarse triangular mesh than with a fine square mesh, and so on'. Wittgenstein is evidently employing this analogy to suggest that an (axiomatized) formulation of mechanics can be thought of as corresponding to a choice of one such net, and that, in this kind of way, mechanics imposes a unified form of description on the world: 'all propositions used in the description of the world must be obtained in a given way from a given set of propositions – the axioms of mechanics'.

Some of the implications of this analogy are spelled out at 6.342:[20]

> The possibility of describing a picture like the ones mentioned above with a net with a given form tells us nothing about the picture. (For that is true of all such pictures.) But what *does* characterize the picture is that it can be described *completely* by a particular net with a *particular* size of mesh.
>
> Similarly the possibility of describing the world by means of Newtonian mechanics tells us nothing about the world: but what does tell us something about it is the precise *way* in which it is possible to describe it by these means. We are also told something about the world by the fact that it can be described more simply with one system of mechanics than with another.

These remarks are again straightforward enough when considering the analogy itself. Whatever the picture formed by the black spots on the white surface, we can describe it by using a sufficiently fine square, triangular, or hexagonal net. Hence, the mere possibility of being able to

[20] 6.342 begins: 'And now we can see the relative position of logic and mechanics. (The net might also consist of more than one kind of mesh: e.g. we could use both triangles and hexagons.)' The relative position of logic and mechanics is something which I discuss only very briefly in this chapter (see Section 4), but it is obviously of crucial importance for connecting Wittgenstein's remarks in the 6.3s with broader themes in the *Tractatus*.

do this does not reveal anything informative about a particular picture. In contrast, it would be informative to know that a particular picture can be 'described *completely* by a particular net with a *particular* size of mesh'. This is informative precisely because the vast majority of possible pictures would *not* be completely (i.e., perfectly) described by that same net with that same size of mesh. So far so good. Applying this back to mechanics, then, Wittgenstein is claiming that the mere possibility of giving a mechanical description of the world is uninformative. What is informative, by contrast, is 'the precise *way* in which it is possible to describe it by these means'. Furthermore, we are told something about the world 'by the fact that it can be described more simply with one system of mechanics than with another'.

Only a handful of commentators have attempted to interpret Wittgenstein's net analogy.[21] Among those, most have suggested that the different nets should be thought of as different coordinate systems. But if this is the correct approach to understanding Wittgenstein's remarks, the analogy would seem to be a very limited one. A central component of the analogy is that a coarser net might nevertheless provide a more accurate description of the picture on the surface. In the case of coordinate systems, however, any one of them can be used to any arbitrary degree of accuracy. Hence, the point of the analogy seems to have been lost entirely.[22] More would need to be said to properly evaluate the existing interpretations of the net analogy, but for present purposes the point of most relevance is that these interpretations do not draw out the connection with Hertz.[23]

With Hertz's comparison of the different formulations of mechanics in mind, the following approach to interpreting 6.341 suggests itself: the three different nets – square, triangular, and hexagonal – can be thought of in comparison with the three different formulations – traditional, energetics, and Hertzian. Indeed, this approach is immediately prompted by Wittgenstein's remark that we are 'told something about the world by the fact that it can be described more simply with one system of mechanics than with another' (6.342). Furthermore, when Wittgenstein says that the use of one of the nets might be simpler than the use of another, this can be

[21] I am aware of interpretations by Black (1964), Griffin (1964), Barker (1979), Hyder (2002), and Tejedor (2015).

[22] Barker (1979) incorporates this point in a sustained criticism of Black's and Griffin's interpretations. It is noteworthy that Barker's article has been very much neglected in more recent literature.

[23] Barker (1979) and Hyder (2002) are the two commentators who take Hertz's influence most seriously.

thought of in comparison to Hertz's notion of one formulation of mechanics being more *distinct* than another.[24] Recall Hertz's account of one of the main advantages of the energetics formulation over the traditional formulation. Instead of *stipulating* that forces are conservative – something that is not strictly required by Newton's laws themselves – the forces that are derived within the energetics formulation, starting from Hamilton's principle, are *automatically* conservative. Insofar as mechanical forces are, in fact, conservative (they obey conservation of energy, only depend on relative distances, and so on), the energetics formulation is therefore more distinct than the traditional formulation. Returning to the net analogy, the need to introduce additional stipulations when using a particular formulation of mechanics would correspond to the need to employ a finer mesh when using a particular net. This would then be a clear example of the kind of thing Wittgenstein had in mind when writing 'it might be that we could describe the surface more accurately with a coarse triangular mesh than with a fine square mesh'.

A more challenging interpretive task awaits us with Wittgenstein's claim that the mere possibility of describing the world with mechanics is uninformative. Mechanics is typically regarded as a substantive theory with substantive content, so that the possibility (or impossibility) of describing the world with it would certainly be informative. On this point, we can begin with the following observation about theory choice quite generally. It is now commonplace that scientific theories are never straightforwardly falsified and that individual scientists are never logically compelled to accept one theory over another.[25] Rather, a new theory is adopted when a large enough portion of the relevant scientific community becomes persuaded by the elegance of the new theory against the clumsiness of the old. If we acknowledge that a certain number of stipulations, not to mention *ad hoc* assumptions, are involved in any theory, then we have a relatively straightforward way to begin to interpret Wittgenstein's remarks at 6.342. For if the introduction of stipulations is unrestricted, it may well be uninformative to be told that a theory *can* be made to fit the facts. (Perhaps *any* theory can be made to fit the facts if massaged enough.) What is informative, by contrast, is the *way* a theory fits the facts. Mechanics, for

[24] The different formulations' relative simplicity (in Hertz's sense) may also be relevant here, but it would obscure the issue to delve into the details on this point. For Hertz, an image is simpler if it contains fewer unnecessary elements, and it is not immediately clear how this would be captured in Wittgenstein's net analogy.

[25] Such a view has become widespread at least since Thomas Kuhn's *Structure of Scientific Revolutions* (Kuhn 2012), first published in 1962, with roots in the work of Quine and Duhem.

instance, is a truly impressive theory (on any formulation) because it can accommodate so many phenomena on the basis of so few fundamental propositions.

Note, however, that there is nothing specific to mechanics in the account of theory choice just outlined – the same considerations would apply to any theory in any of the sciences. But mechanics is not like any scientific theory: It is 'an attempt to construct according to a single plan' all the propositions needed 'for a description of the world' (6.343). In other words, mechanics is (or purports to be) a *fundamental* theory. At its root, mechanics is a theory of matter and motion, a completely general theory of the behavior of objects in space and time. Mechanics can thus be seen to underlie every other kind of scientific theory; theories that concern some more specific behavior of objects in space and time. As Wittgenstein notes, however, mechanics itself is only one 'attempt' (*Versuch*) to construct a fundamental theory (6.343), an attempt that was supplanted, in an important sense, by the theory of relativity. As Wittgenstein is seeing things, all such attempts – all candidate fundamental theories – impose a unified form on the description of the world; they all 'supply the bricks for building the edifice of science' (6.341).

On this point, it is instructive to think of Newton's discussion of the notions of space and time in *De Gravitatione* and later in *Principia*.[26] Here it emerges that the core motivation feeding into Newton's definitions of 'absolute' space and time stems from the need to articulate coherent definitions of the basic kinematical notions: determinate positions, trajectories, velocities, and so on. Although Newton overstepped insofar as his notion of absolute rest is not actually required for mechanics,[27] Newton's first law of motion is now typically understood as a claim about the inertial structure of spacetime: the first law implies that there is a family of inertial frames of reference from the perspective of which there is agreement concerning which bodies in the universe are moving with a constant velocity. Importantly, specifying an inertial structure in this way is fundamental for any further physics. If we are not in a position to identify definite locations or velocities, we cannot make sense of any particular force laws (such as the law of gravitational attraction). In fact, any scientific description of phenomena whatsoever relies, at least implicitly, on some

[26] For English translations, see Newton (2004: §II and §III).

[27] Using contemporary mathematical resources, we distinguish between 'Newtonian' spacetime, with a full commitment to the preferred frame of reference that absolute space implies, and 'Galilean' spacetime, which commits only to a family of inertial frames. When contemporary philosophers speak of the spacetime of classical mechanics, they typically mean Galilean spacetime.

sort of spatio-temporal framework. In this way, mechanics, as a fundamental theory, is a theory *within which* other empirical propositions can be articulated.[28]

This points to a way of interpreting 6.342 that does involve specific features of mechanics, in particular, an interpretation that involves the conception of mechanics as a fundamental theory. When Wittgenstein says 'the possibility of describing the world by means of Newtonian mechanics tells us nothing about the world', we can interpret this as claiming that it is always possible (in a fairly weak sense) to describe phenomena using the resources of classical physics, including, in particular, the classical notions of space and time. Of course, in the face of what we now regard as relativistic phenomena (not to mention quantum phenomena), it is no longer appealing to do so. But this is a fact that Wittgenstein's account can readily accommodate: We are told something about the world by the *way* it is possible to describe it by using a classical or nonclassical framework; we are told something by the fact that we achieve, or fail to achieve, an elegant uniformity in our descriptions of nature.

Let us now turn, finally, to Wittgenstein's explicit reference to Hertz at 6.361.

9.3.2 Only Normal Connections Are Thinkable

At a first pass, Wittgenstein's claim that only normal connections are thinkable is very puzzling. Merely by defining normal connections as independent of time and abnormal connections as dependent on time, both kinds of connections appear to be at least *thinkable* already. Furthermore, we have a whole class of systems – guided systems – that have abnormal connections. This is why Hertz shows that, even though the fundamental law cannot be applied directly to a system with abnormal connections, a number of other principles (such as the principle of least acceleration) can. Far from being unthinkable, then, we appear to be able to work successfully with abnormal connections within Hertz's mechanics.

However, as we have seen, the abnormal connections of guided systems are in an important sense only apparent. In the final analysis, a guided system is a partial system, and the complete system of which it is a part has

[28] The same can be said of the theories of both special and general relativity, each of which articulates more sophisticated spatio-temporal frameworks. For some relevant discussion in this direction, see Friedman (2001: 79–81) and DiSalle (1995).

only normal connections. Indeed, Hertz writes that 'if, owing to any particular form of the equations of condition, this assumption is not permissible, then these equations of condition already involve a contradiction to the fundamental law or its assumptions, and no questions asked concerning the system would be mechanical problems' (§436). In other words, if we had before us a *fundamental* abnormal connection, then we would have left the realm of mechanics altogether.

Let us grant, then, that it is this kind of connection – a connection that depends on time fundamentally – which is unthinkable according to 6.361. Even in these kinds of cases, however, at least Hertz himself does not seem to regard such systems as inconceivable. Hertz is circumspect regarding the potential universal validity of mechanics and takes care to note that the theory certainly does not seem to apply to biological systems. (On Hertz's view, to claim that living creatures are governed by mechanics has the character of a *permissible* but *improbable* hypothesis; see §§319–320.) For Hertz, then, the fundamental law can serve as a demarcation between mechanical and nonmechanical phenomena: Systems that violate the fundamental law are simply systems that are not described by mechanics. Although such a view may strike us as a perfectly reasonable one, it is evidently not Wittgenstein's view. For Wittgenstein, the occurrence of a fundamental abnormal connection would not indicate that we are dealing with a nonmechanical phenomenon; such a connection is *unthinkable*. Now, why would Wittgenstein claim that?

Let us begin to attempt to answer this question by turning to the immediate comment on 6.361:

> 6.3611 We cannot compare a process with the 'passage of time' – there is no such thing – but only with another process (such as the working of a chronometer).
>
> Hence we can describe the lapse of time only by relying on some other process. Something exactly analogous applies to space: e.g. when people say that neither of two events (which exclude one another) can occur, because there is *nothing to cause* the one rather than the other, it is really a matter of being unable to describe *one* of the two events unless there is some sort of asymmetry to be found. And *if* such an asymmetry *is* to be found, we can regard it as the *cause* of the occurrence of the one and the non-occurrence of the other.

The opening sentences of 6.3611 suggest what might appear to be a way to make sense of 6.361. Perhaps a system with abnormal connections is unthinkable because we would seem to be comparing such a system with the passage of time itself, and that would be impossible if there is no such thing as the passage of time. However, there is an immediate problem with this line of interpretation. We would not need to compare an abnormal system to the passage of time (which, of course, we never do); we would only need to compare it to other (normal) systems. In particular, we would only need to compare it to the kinds of systems that we use as clocks.

To take a different tack, we can note that Wittgenstein's main topic in 6.3611, evident from what he goes on to say in his remarks about space, is the role of certain kinds of asymmetries in our descriptions. A description of a difference between two objects relies, of course, on there being a difference to describe.[29] This applies equally well to a difference across time (e.g., one object accelerating in comparison to another moving at a constant velocity) as well as across space (e.g., one object being larger than another). An eternally homogeneous universe would be a perfect void – there would be no discernible objects to be described at all.[30] In this way, the remarks about temporal and spatial asymmetries in 6.3611 can be recognized as concerned with the possibility of any kind of description at all.

Now, why does Wittgenstein pivot in the final paragraph of 6.3611 to a discussion of causality? Indeed, it is evident that causality takes center stage in the nearby framing remarks. 6.361 is itself the first comment on 6.36:

> 6.36 If there were a law of causality, it might be put the following way: There are laws of nature.
>
> But of course that cannot be said: it makes itself manifest.

Then, following the 6.361s, 6.362 states, 'What can be described can happen too: and what the law of causality is meant to exclude cannot even be described'. Our immediate task is thus to bring this connection between temporal and spatial asymmetries, on the one hand, and the notion of causality, on the other, into focus.

The notion of causality is discussed earlier, at 6.32 and 6.321:[31]

> 6.32 The law of causality is not a law but the form of a law.

[29] Put another way: The possibility of a description relies on the possibility of distinguishing the object in question, otherwise there is nothing to describe. Compare 2.02331, 'if there is nothing to distinguish a thing, I cannot distinguish it, since otherwise it would be distinguished after all'.

[30] Hyder (2002: §6.2.3) presents a detailed discussion of this idea, drawing a connection with Maxwell's characterization of natural laws 'as those independent of time and space' (Hyder 2002: 178).

[31] The first mention of causality in the *Tractatus* is at 5.136–5.1362, but I do not discuss those remarks in this chapter.

> 6.321 'Law of causality' – that is a general name. And just as in mechanics, for example, there are 'minimum-principles', such as the law of least action, so too in physics that are causal laws, laws of the causal form.

In mechanics there are indeed 'minimum-principles', principles that state that some quantity takes a minimum value.[32] Wittgenstein gives the example of the law of least action, and other examples that we have already encountered include the principle of least acceleration and Gauss' principle of least constraint. Indeed, Hertz's fundamental law is a further example: In following a 'straightest' path a free system will be following a path of minimum curvature. There is of course no single, overarching 'minimum-principle'; rather, there are various different principles that share a recognizable form. At 6.33 and 6.34, Wittgenstein mentions further 'forms in which the propositions of science can be cast', the most straightforward example of which is the form of a conservation law. Just as there are various minimum principles, so too there are various conservation principles – conservation of mass or momentum or energy, and so on. Here again we have a group of principles that share a recognizable form. Returning to the notion of causality: when Wittgenstein writes that the law of causality 'is not a law but the form of a law', he is claiming that, in an analogous way, although there are different *laws of the causal form*, there is no single, overarching, law of causality. Note, however, that the causal form is a special case because it operates at a higher degree of generality: 'If there were a law of causality, it might be put in the following form: There are laws of nature' (6.36). The suggestion here is that the so-called law of causality is best understood as an attempt to capture the common form of any and all natural laws. But what could that common form be?

Recall the discussion of temporal and spatial asymmetries at 6.3611:

> [W]hen people say that neither of two events (which exclude one another) can occur, because there is *nothing to cause* the one rather than the other, it is really a matter of being unable to describe *one* of the two events unless there is some sort of asymmetry to be found. And *if* such an asymmetry *is* to be found, we can regard it as the *cause* of the occurrence of the one and the non-occurrence of the other.

This provides a hint for the common form of natural laws: descriptions of events in terms of spatial and temporal asymmetries. One might then be

[32] So-called minimum principles are often better described as 'extremal' principles – the quantity in question is typically required to be a (local) minimum, maximum, or point of inflection.

tempted to call such descriptions 'causal', but we must note that they are really just descriptions *simpliciter*. The supposed contrast class of 'non-causal' descriptions would be made up of attempts to describe differences where there aren't any. If we did not employ any temporal or spatial asymmetries in our descriptions, we would not be describing anything. (If there were no temporal or spatial asymmetries in our world, then our world would be eternally homogeneous.) Thus, Wittgenstein's remark, 'What can be described can happen too: and what the law of causality is meant to exclude cannot even be described' (6.362).

We are now, finally, in a position to understand Wittgenstein's elliptical remark about the unthinkability of abnormal connections. According to 6.341, mechanics 'imposes a unified form of description on the world' such that 'all propositions used in the description of the world must be obtained in a given way from a given set of propositions – the axioms of mechanics'. This applies equally to any adequately axiomatized formulation of mechanics, but with reference to Hertz's formulation in particular, all propositions used in the description of the world would be obtained from just a single axiom: the fundamental law. On Wittgenstein's view, classical mechanics, as a fundamental theory, is an attempt to situate all true descriptions of nature within a single, overarching system. On that understanding, violations of the fundamental law would not simply indicate the limited scope of the theory. Unless and until the framework of mechanics is replaced by some alternative framework, to violate the fundamental law is to step outside the realm of ('causal') descriptions altogether. As we have seen, a fundamental abnormal connection would represent a violation of the fundamental law, hence a violation of the bounds of what can be described. It is in this sense, then, that only normal connections – only lawlike (*gesetzmäßig*) connections – are thinkable.

9.4 Conclusion

This chapter has been oriented around 6.361 and Wittgenstein's second and final reference to Hertz in the *Tractatus*.[33] We have seen that a satisfactory interpretation of this remark requires detailed attention to Hertz's reformulation of mechanics in *Principles*, for it is only with this in hand that we can understand why Wittgenstein would claim that only normal connections are thinkable. Furthermore, we have seen that attention to Hertz's work makes many of Wittgenstein's remarks in the

[33] For a discussion of Wittgenstein's first reference to Hertz at 4.04, see Eisenthal (2022).

6.3s significantly less obscure than they might otherwise appear, particularly where they concern mechanics. One concrete result of this discussion is an insight into why Wittgenstein links the notion of causality to the common form of all natural laws, and why 'what the law of causality is meant to exclude cannot even be described'.

Of the many closely connected issues that warrant further discussion, one of particular importance relates to the framing remarks of the whole 6.3 sequence, especially 6.3 itself:

> 6.3 The exploration of logic means the exploration of *everything that is subject to law* [*aller Gesetzmäßigkeit*]. And outside logic everything is accidental.

Here we have a nominalization of the very same word that occurs in 6.361 – *gesetzmäßig*, 'subject to law' – the word that has been at the center of much of the discussion of this chapter. But note that the notion of law that is operative at 6.3, in connection with logic, appears quite different to the notion of law that is operative within the 6.3 sequence, in connection with the natural sciences. I have suggested that the common form of all natural laws is the appeal to descriptions of events in terms of spatial and temporal asymmetries, but this, as we have seen, collapses down to descriptions of events *simpliciter*. The crucial point here is that a substantive notion of natural law – of lawlikeness in nature – seems to have dropped out of the picture. This connects with one of the leading ideas in the *Tractatus*: the claim that the modal notions of necessity and contingency *only* find their significance in logic.[34] As 6.37 puts it, 'There is no compulsion making one thing happen because another thing has happened. The only necessity that exists is *logical* necessity'. The thought here, in line with the thought at 6.3, is that natural laws do not have the kind of *causal significance* that we typically take them to have. A proper discussion of this point, however, will have to wait for another occasion.

[34] Compare 5.525: 'The certainty, possibility, or impossibility of a situation is not expressed by a proposition, but by an expression's being a tautology, a proposition with a sense, or a contradiction'.

WITTGENSTEIN'S *TRACTATUS LOGICO-PHILOSOPHICUS*

A Critical Guide

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References

Anscombe, G. E. M. 1957. *Intention*. Oxford: Blackwell.

1971. *An Introduction to Wittgenstein's Tractatus*. 2nd ed. London: Hutchinson.

Anzengruber, Ludwig. 1872. "Die Kreuzelschreiber". In *Ludwig Anzengrubers Gesammelte Werke*. Stuttgart and Berlin: G. Cotta'sche Buchhandlung Nachfolger.

Appelqvist, Hanne. 2016. "On Wittgenstein's Kantian Solution of the Problem of Philosophy". *The British Journal for the History of Philosophy* 24 (4):697–719.

Aristotle. 1964. *Analytica Priora*. Edited by W. D. Ross. Oxford: Clarendon Press.

Barker, Peter. 1979. "Untangling the Net Metaphor". *Philosophy Research Archives* 5:182–199.

Bell, David. 1996. "Solipsism and Subjectivity". *European Journal of Philosophy* 4:155–174.

Black, Max. 1964. *A Companion to Wittgenstein's Tractatus*. Ithaca, NY: Cornell University Press.

Bradley, Raymond. 1992. *The Nature of All Being: A Study of Wittgenstein's Modal Atomism*. New York and Oxford: Oxford University Press.

Burnyeat, Myles. 1987. "Wittgenstein and Augustine De Magistro". *Proceedings of the Aristotelian Society Supplementary Volume* 61:1–24.

Butterfield, Jeremy. 2004. Between Laws and Models: Some Philosophical Morals of Lagrangian Mechanics, https://arxiv.org/pdf/physics/0409030.pdf.

Carnap, Rudolf. 1930. "Die Alte Und Die Neue Logik". *Erkenntnis* 1 (1):12–26.

1983. "The Logicist Foundations of Mathematics". In *Philosophy of Mathematics: Selected Readings*, edited by P. Benacerraf and H. Putnam. Cambridge: Cambridge University Press, 41–52.

Cartwright, Richard. 1987. *Philosophical Essays*. Cambridge, MA: MIT Press.

2003. "Russell and Moore, 1898–1905". In *The Cambridge Companion to Bertrand Russell*, edited by N. Griffin. Cambridge: Cambridge University Press, 108–127.

Child, William. 2013. "Does the Tractatus Contain a Private Language Argument?". In *Wittgenstein's Tractatus: History and Interpretation*, edited by P. Sullivan and M. Potter. Oxford: Oxford University Press, 143–169.

Christensen, Anne-Marie Søndergaard. 2011. "Wittgenstein and Ethics". In *The Oxford Handbook of Wittgenstein*, edited by O. Kuusela and M. McGinn. Oxford: Oxford University Press, 796–817.

Church, Alonzo. 1932. "A Set of Postulates for the Foundation of Logic". *The Annals of Mathematics* 33 (2):346–366.

1941. *The Calculi of Lambda Conversion*. Princeton, NJ: Princeton University Press.

Chwistek, Leon. 1967. "Antinomies of Formal Logic". In *Polish Logic: 1920–1939*, edited by S. McCall. Oxford: Clarendon Press, 338–345.

Conant, James. 2000. "Elucidation and Nonsense in Frege and Early Wittgenstein". In *The New Wittgenstein*, edited by A. Crary and R. Read. London: Routledge, 174–217.

2002. "The Method of the Tractatus". In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by E. H. Reck. Oxford: Oxford University Press, 374–462.

2011. "Wittgenstein's Methods". In *The Oxford Handbook of Metaphysics*, edited by O. Kuusela and M. McGinn. Oxford: Oxford University Press, 620–645.

Copi, Irving M. 1958. "Objects, Properties and Relations in the Tractatus". *Mind* 67:145–165.

Diamond, Cora. 1991a. "The Face of Necessity". In *The Realistic Spirit*. Cambridge, MA: MIT Press, 243–266.

1991b. "Throwing Away the Ladder: How to Read the *Tractatus*". In *The Realistic Spirit*. Cambridge, MA: MIT Press, 179–204.

1991c. "What Nonsense Might Be". In *The Realistic Spirit*. Cambridge, MA: MIT Press, 95–114.

2000a. "Does Bismarck Have a Beetle in His Box? The Private Language Argument in the Tractatus". In *The New Wittgenstein*, edited by A. Crary and R. Read. Abingdon: Routledge, 262–292.

2000b. "Ethics, Imagination and the Method of Wittgenstein's *Tractatus*". In *The New Wittgenstein*, edited by A. Crary and R. Read. Abingdon: Routledge, 149–173.

2002. "Truth before Tarski: After Sluga, after Ricketts, after Geach, after Goldfarb, Hylton, Floyd, and Van Heijenoort". In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by E. H. Reck. Oxford: Oxford University Press, 252–279.

2018. "Commentary on José Zalabardo's 'the Tractatus on Unity'". *Australasian Philosophical Review* 2:272–284.

2019. *Reading Wittgenstein with Anscombe, Going on to Ethics*. Cambridge, MA: Harvard University Press.

DiSalle, Robert. 1995. "Spacetime Theory as Physical Geometry". *Erkenntnis* 42 (3):317–337.

Dreben, Burton, and Juliet Floyd. 1991. "Tautology: How Not to Use a Word". *Synthese* 87 (1):23–50.

Drury, Maurice O'Connor. 1981. "Some Notes on Conversations with Wittgenstein". In *Ludwig Wittgenstein. Personal Recollections*, edited by R. Rhees. Totowa, NJ: Rowman and Littlefield, 76–96.

Dummett, Michael. 1959. "Truth". *Proceedings of the Aristotelian Society* 59:141–162.

1991. "Frege's Myth of the Third Realm". In *Frege and Other Philosophers*. Oxford: Oxford University Press, 249–262.

Eisenthal, Joshua. 2018. "Mechanics without Mechanisms". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 62:42–55.

2021. "Hertz's Mechanics and a Unitary Notion of Force". *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 90:226–234.

2022. "Models and Multiplicities". *Journal of the History of Philosophy* 60 (2):277–302.

Engelmann, Mauro Luiz. 2021. *Reading Wittgenstein's Tractatus*. Cambridge: Cambridge University Press.

Fairhurst, Jordi. 2019. "The Ethical Subject and the Willing Subject in the Tractatus: An Alternative to the Transcendental Reading". *Philosophia* 47 (1):75–95.

Floyd, Juliet. 1995. "On Saying What You Really Want to Say: Wittgenstein, Gödel, and the Trisection of the Angle". In *From Dedekind to Gödel: Essays on the Development of the Foundations of Mathematics*, edited by J. Hintikka. Dordrecht: Kluwer 373–425.

1998. "The Uncaptive Eye: Solipsism in Wittgenstein's Tractatus". In *Loneliness*, edited by L. S. Rouner. Notre Dame, IN: University of Notre Dame Press, 79–108.

2000. "Wittgenstein, Mathematics and Philosophy". In *The New Wittgenstein*, edited by R. Read and A. Crary. London: Routledge, 232–261.

2001. "Number and Ascriptions of Number in the Tractatus". In *Future Pasts: Perspectives on the Analytic Tradition in Twentieth Century Philosophy*, edited by J. Floyd and S. Shieh. New York: Oxford University Press, 145–191.

2005. "Wittgenstein on Philosophy of Logic and Mathematics". In *The Oxford Handbook to the Philosophy of Logic and Mathematics*, edited by S. Shapiro. New York: Oxford University Press, 75–128.

2007. "Wittgenstein and the Inexpressible". In *Wittgenstein and the Moral Life*, edited by A. Crary. Cambridge, MA: MIT Press, 177–234.

2010. "On Being Surprised: Wittgenstein on Aspect Perception, Logic and Mathematics". In *Seeing Wittgenstein Anew: New Essays on Aspect Seeing*, edited by W. Day and V. J. Krebs. New York: Cambridge University Press, 314–337.

2018. "'Ultimate' Facts? Zalabardo on the Metaphysics of Truth". *Australasian Philosophical Review* 2 (3):299–314.

Fogelin, Robert J. 1987. *Wittgenstein*. 2nd ed. London: Routledge and Kegan Paul.

Frascolla, Pasquale. 1994. *Wittgenstein's Philosophy of Mathematics*. London: Routledge.

1997. "The Tractatus System of Arithmetic". *Synthese* 122:353–378.

2004. "On the Nature of Tractatus Objects". *Dialectica* 58 (3):369–382.

2007. *Understanding Wittgenstein's Tractatus*. London: Routledge.

2016. “Wittgenstein’s Early Philosophy of Mathematics”. In *A Companion to Wittgenstein*, edited by H.-J. Glock and J. Hyman. Hoboken, NJ: Wiley-Blackwell, 305–318.

Fraser, Craig. 1985. “D’Alembert’s Principle: The Original Formulation and Application in Jean D’Alembert’s Traité de Dynamique (1743)”. *Centaurus* 28:31–61.

Frege, Gottlob. 1879. *Begriffsschrift, Eine Der Arithmetischen Nachgebildete Formelsprache Des Reinen Denkens*. Halle: L. Nebert.

1893. *Grundgesetze Der Arithmetik*. Vol. I. Jena: Verlag Hermann Pohle.

1906. “Über Die Grundlagen Der Geometrie I, II, III”. *Jahresbericht der Deutschen Mathematiker-Vereinigung* 15:293–309, 377–403, 423–430.

1972. *Conceptual Notation and Related Articles*. Translated by T. W. Bynum. Edited by T. W. Bynum. Oxford: Clarendon Press.

1979a. “Logic in Mathematics”. In *Posthumous Writings*, edited by H. Hermes, F. Kambartel and F. Kaulbach. Oxford: Blackwell, 203–250.

1979b. *Posthumous Writings*. Translated by P. Long and R. White. Edited by H. Hermes, F. Kambartel and F. Kaulbach. Oxford: Blackwell.

1980a. *The Foundations of Arithmetic*. Translated by J. L. Austin. Evanston, IL: Northwestern University Press.

1980b. “On Concept and Object”. In *Translations from the Philosophical Writings of Gottlob Frege*, edited by M. Black and P. T. Geach. Oxford: Basil Blackwell, 42–55.

1984. *Collected Papers on Mathematics, Logic, and Philosophy*. Edited by B. McGuinness. Oxford: Blackwell.

2013. *Basic Laws of Arithmetic*. Edited by P. Ebert and M. Rossberg. Oxford: Oxford University Press.

Friedlander, Eli. 2017. “Logic, Ethics and Existence in Wittgenstein’s *Tractatus*”. In *Wittgenstein’s Moral Thought*, edited by E. Dain and R. Agam-Segal. London: Routledge, 97–131.

Friedman, Michael. 2001. *Dynamics of Reason: The 1999 Kant Lectures at Stanford University*. Stanford, CA: CSLI Publications.

Gabriel, Gottfried. 2017. “Solipsism – The Leitmotif in Wittgenstein’s Life and Philosophy”. *Wittgenstein Studien* 8:1–14.

Gandon, Sébastien. 2002. *Logique Et Langage. Études Sur Le Premier Wittgenstein*. Paris: Vrin.

Georgallides, Andreas. 2021. “A Possible Resolution of the Tractarian Paradox”. *Open Journal of Philosophy* 11 (1):148–158.

Glock, Hans-Johann. 1992. “Cambridge, Jena or Vienna? The Roots of the *Tractatus*”. *Ratio* 5 (1):1–23.

1997. “Kant and Wittgenstein: Philosophy, Necessity and Representation”. *International Journal of Philosophical Studies* 5 (2):285–305.

Goldfarb, Warren. 1997. “Metaphysics and Nonsense: On Cora Diamond’s The Realistic Spirit”. *Journal of Philosophical Research* 22:57–74.

Gómez-Alonso, Modesto. 2016. “Wittgenstein on the Will and Voluntary Action”. In *Action, Decision-Making and Forms of Life*, edited by J. Padilla Gálvez. Berlin and Boston, MA: De Gruyter, 77–108.

Griffin, James. 1964. *Wittgenstein's Logical Atomism*. Oxford: Oxford University Press.

Griffin, Nicholas. 1985. "Russell's Multiple Relation Theory of Judgement". *Philosophical Studies* 47:213–248.

ed. 1992. *The Selected Letters of Bertrand Russell, Vol. 1: The Private Years (1884–1914)*. Boston, MA: Houghton Mifflin Co.

Hacker, P. M. S. 1972. *Insight and Illusion: Wittgenstein on Philosophy and the Metaphysics of Experience*: Oxford: Clarendon Press.

1986. *Insight and Illusion: Themes in the Philosophy of Wittgenstein*. Rev. ed. Oxford: Clarendon.

Hancock, Peter, and Per Martin-Löf. 1975. Syntax and Semantics of the Language of Primitive Recursive Functions. In *Research Reports in Mathematics*. Preprint 3. Stockholm: Department of Mathematics, University of Stockholm.

Hanslick, Eduard. 1986. *On the Musically Beautiful*. Translated by G. Payzant. Indianapolis: Hackett.

Hertz, Heinrich. 1894. *Prinzipien Der Mechanik*. Leipzig: J.A. Barth.

1899. *The Principles of Mechanics Presented in a New Form*. London: Macmillan and Co.

Hume, David. 1978. *A Treatise of Human Nature*. 2nd ed. Oxford: Clarendon.

Hyder, David Jalal. 2002. *The Mechanics of Meaning: Propositional Content and the Logical Space of Wittgenstein's "Tractatus"*. Berlin: de Gruyter.

Hylton, Peter. 1990. *Russell, Idealism, and the Emergence of Analytic Philosophy*. Oxford: Clarendon.

Janik, Allan. 2000. *Wittgenstein's Vienna Revisited*. New Brunswick: Transaction Publishers.

Kannisto, Heikki. 1986. *Thoughts and Their Subject: A Study of Wittgenstein's Tractatus*. Helsinki: Philosophical Society of Finland.

Kant, Immanuel. 1977. *Prolegomena to Any Future Metaphysics*. Translated by P. Carus and J. W. Ellington. Indianapolis: Hackett.

1998. *Critique of Pure Reason*. Translated by P. Guyer and A. Wood. Edited by P. Guyer and A. Wood. Cambridge: Cambridge University Press.

2000. *Critique of the Power of Judgment*. Translated by P. Guyer and E. Matthews. Edited by P. Guyer. Cambridge: Cambridge University Press.

2006. *Anthropology from a Pragmatic Point of View*. Translated by R. B. Louden. Edited by R. B. Louden. Cambridge: Cambridge University Press.

Kimhi, Irad. 2018. *Thinking and Being*. Cambridge, MA: Harvard University Press.

Kjærgaard, Peter C. 2002. "Hertz and Wittgenstein's Philosophy of Science". *Journal for General Philosophy of Science* 33 (1):121–149.

Kremer, Michael. 2001. "The Purpose of Tractarian Nonsense". *Noûs* 35 (1):39–73.

2002. "Mathematics and Meaning in the *Tractatus*". *Philosophical Investigations* 25:272–303.

2004. "To What Extent Is Solipsism a Truth?". In *Post-Analytic Tractatus*, edited by B. Stocker. London: Routledge, 59–84.

2007. "The Cardinal Problem of Philosophy". In *Wittgenstein and the Moral Life. Essays in Honour of Cora Diamond*, edited by A. Crary. Cambridge, MA: MIT Press, 143–176.

2012. "Russell's Merit". In *Wittgenstein's Early Philosophy*, edited by J. L. Zalabardo. Oxford: Oxford University Press, 195–241.

Kuhn, Thomas S. 2012. *The Structure of Scientific Revolutions*. Chicago, IL: University of Chicago Press.

Kuusela, Oskari. 2011. "The Development of Wittgenstein's Philosophy". In *The Oxford Handbook of Wittgenstein*, edited by O. Kuusela and M. McGinn. Oxford: Oxford University Press, 597–619.

2017. "Wittgenstein, Ethics and Philosophical Clarification". In *Wittgenstein's Moral Thought*, edited by E. Dain and R. Agam-Segal. London: Routledge, 37–65.

2019. *Wittgenstein on Logic as the Method of Philosophy. Re-Examining the Roots and Development of Analytic Philosophy*. Oxford: Oxford University Press.

2023. "Wittgenstein's *Tractatus* and the Epistemology of Logic". In *Wittgenstein's Tractatus at 100*, edited by M. Stokhof and H. Tang. Cham, Switzerland: Palgrave Macmillan, 35–56.

Lanczos, Cornelius. 1962. *The Variational Principles of Mechanics*. Toronto: University of Toronto Press.

Lebens, Samuel. 2017. *Bertrand Russell and the Nature of Propositions*. London: Routledge.

Lewy, C. 1967. "A Note on the Text of the *Tractatus*". *Mind* 76:416–423.

Longuenesse, Béatrice. 2017. *I, Me, Mine; Back to Kant, and Back Again*. Oxford: Oxford University Press.

Lützen, Jesper. 2005. *Mechanistic Images in Geometric Form: Heinrich Hertz's 'Principles of Mechanics'*. Oxford: Oxford University Press.

MacBride, Fraser. 2013. "The Russell-Wittgenstein Dispute: A New Perspective". In *Judgement and Truth in Early Analytic Philosophy and Phenomenology*, edited by M. Textor. Basingstoke: Palgrave Macmillan, 206–241.

Malcolm, Norman. 1984. *Ludwig Wittgenstein. A Memoir with a Biographical Sketch by G.H. Von Wright and Wittgenstein's Letters to Malcolm*. Oxford: Oxford University Press.

Marion, Mathieu. 1998. *Wittgenstein's Finitism and the Foundations of Mathematics*. Oxford: Clarendon Press.

2011. "Wittgenstein on the Surveyability of Proofs". In *The Oxford Handbook on Wittgenstein*, edited by M. McGinn and O. Kuusela. Oxford: Oxford University Press, 138–161.

Marion, Mathieu, and Mitsuhiro Okada. 2018. "Wittgenstein, Goodstein and the Origin of the Uniqueness Rule for Primitive Recursive Arithmetic". In *Wittgenstein in the 1930s. Between the Tractatus and the Investigations*, edited by D. Stern. Cambridge: Cambridge University Press, 253–271.

McGinn, Marie. 2006. *Elucidating the Tractatus: Wittgenstein's Early Philosophy of Language and Logic*. Oxford: Clarendon.

McGuinness, Brian. 1966. "The Mysticism of the Tractatus". *Philosophical Review* 75 (3):305–328.

2002a. *Approaches to Wittgenstein. Collected Papers*. London: Routledge.

2002b. "Solipsism". In *Approaches to Wittgenstein. Collected Papers*. London: Routledge, 131–139.

2005. *Young Ludwig: Wittgenstein's Life, 1889–1921*. Oxford: Oxford University Press.

2006. "Wittgenstein and Ramsey". In *Cambridge and Vienna: Frank P. Ramsey and the Vienna Circle*, edited by M. C. Galavotti. Vienna and New York: Springer, 19–28.

ed. 2008. *Wittgenstein in Cambridge. Letters and Documents 1911–1951*. Oxford: Blackwell.

McGuinness, Brian, and G. H. von Wright, eds. 1995. *Ludwig Wittgenstein: Cambridge Letters*. Oxford: Basil Blackwell.

Monk, Ray. 1991. *Ludwig Wittgenstein: The Duty of Genius*. London: Vintage.

Moore, Adrian W. 1987. "On Saying and Showing". *Philosophy* 62 (242):473–497.

2003. "Ineffability and Nonsense". *Aristotelian Society Supplementary Volume* 77 (1):169–193.

2013. "Was the Author of the Tractatus a Transcendental Idealist". In *Wittgenstein's Tractatus: History and Interpretation*, edited by P. Sullivan and M. Potter. Oxford: Oxford University Press, 239–255.

2020. "The Bounds of Nonsense". In *Wittgenstein and the Limits of Language*, edited by H. Appelqvist. New York: Routledge, 27–45.

Moore, G. E. 1901. "Truth and Falsity". In *Dictionary of Philosophy and Psychology*, edited by J. M. Baldwin. New York: The Macmillan Company, 716–18.

Munz, Regine. 1995. "Wittgensteins Ehrfurcht". In *Culture and Value / Philosophie Und Die Kulturwissenschaften. Pre-Proceedings of the 18th International Wittgenstein Symposium in Kirchberg Am Wechsel*, edited by K. S. Johannessen and T. Nordenstam. Vienna: Hölder-Pichler-Tempsky, 68–74.

Myhill, John. 1974. "The Undefinability of the Set of Natural Numbers in the Ramified Principia". In *Bertrand Russell's Philosophy*, edited by G. Nakhnikian. London: Duckworth, 19–27.

Narboux, Jean-Philippe. 2009. "Négation et totalité dans Le Tractatus De Wittgenstein". In *Lire Le Tractatus De Wittgenstein*, edited by C. Chauviré. Paris: Vrin, 127–176.

2014. "Showing, the Medium Voice, and the Unity of the Tractatus". *Philosophical Topics* 42 (2):201–262.

Newton, Isaac. 2004. *Issac Newton: Philosophical Writings*. Cambridge: Cambridge University Press.

Nir, Gilad. 2021. "The Tractatus and the Riddles of Philosophy". *Philosophical Investigations* 44 19–42.

Okada, Mitsuhiro. 2007. "On Wittgenstein's Remarks on Recursive Proofs: A Preliminary Report". In *Essays in the Foundations of Logical and*

Phenomenological Studies, edited by M. Okada. Tokyo: Keio University Press, 121–131.
Oz, Amos. 2003. *Sipour Al Ahava Vehoshekh [A Tale of Love and Darkness]*. Jerusalem: Keter Publishing House.
2017. *A Tale of Love and Darkness*. Translated by N. de Lange. London: Vintage.
Pascal, Blaise. 1958. *Pensées*. New York: E. P. Dutton & Co.
Peano, Giuseppe. 1967. "The Principles of Arithmetic, Presented by a New Method". In *From Frege to Gödel. A Sourcebook in Mathematical Logic. 1879–1931*, edited by J. van Heijenoort. Cambridge, MA: Harvard University Press, 85–97.
Pilch, Martin. 2016. Wittgenstein Source Prototractatus Tools (PTT). In *Wittgensteinsource* (www.Wittgensteinsource.org), edited by A. Pichler and J. Kathrein. Bergen: Wittgenstein Archives, University of Bergen.
Plato. 1985. "Sophista". In *Platonis Opera*, edited by J. Burnet. Oxford: Clarendon Press.
Potter, Michael. 2000. *Reason's Nearest Kin. Philosophies of Arithmetic from Kant to Carnap*. Oxford: Oxford University Press.
2009a. "The Logic of the Tractatus". In *Handbook of the History of Logic. Volume 5. Logic from Russell to Church*, edited by D. M. Gabbay and J. Woods. Amsterdam: North Holland, 255–304.
2009b. *Wittgenstein's Notes on Logic*. Oxford: Oxford University Press.
2020. *The Rise of Analytic Philosophy, 1879–1930: From Frege to Ramsey*. Abingdon, Oxon: Routledge.
2021. "How Substantial Are Tractarian Objects Really?". *Disputatio (Spain)* 10:93–107.
Preston, John. 2008. "Hertz, Wittgenstein and Philosophical Method". *Philosophical Investigations* 31 (1):48–67.
Proops, Ian. 1997. "The Early Wittgenstein on Logical Assertion". *Philosophical Topics* 25 (2):121–144.
Quine, Willard Van Orman. 1960. *Word and Object*. Cambridge, MA: MIT Press.
Ramsey, Frank. 1926. "The Foundations of Mathematics". *Proceedings of the London Mathematical Society* 25:338–384.
Rhees, Rush. 1947. "Review of Cornforth, Science and Idealism". *Mind* 56:374–392.
Richter, Duncan. 2018. "Augustine and Wittgenstein on the Will". In *Augustine and Wittgenstein*, edited by K. Paffenroth, A. R. Eodice and J. Doody. Lanham, MD: Lexington Books, 169–184.
Ricketts, Thomas. 1996. "Pictures, Logic, and the Limits of Sense in Wittgenstein's Tractatus". In *The Cambridge Companion to Wittgenstein*, edited by H. Sluga and D. G. Stern. Cambridge: Cambridge University Press, 59–99.
2002. "Wittgenstein against Frege and Russell". In *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*, edited by E. H. Reck. Oxford: Oxford University Press, 227–251.

Rogers, Brian, and Kai Wehmeier. 2012. "Tractarian First-Order Logic: Identity and the N-Operator". *Review of Symbolic Logic* 5 (4):538–573.

Russell, Bertrand. 1903. *The Principles of Mathematics*. Cambridge: Cambridge University Press.

1904. "Meinong's Theory of Complexes and Assumptions". *Mind* 13:204–219, 336–354, 509–524.

1905. "On Denoting". *Mind* 14:479–493.

1908. "Mathematical Logic as Based on the Theory of Types". *American Journal of Mathematics* 30 (3):222–262.

1912. *The Problems of Philosophy*. London: Williams & Norgate.

1914. *Our Knowledge of the External World*. London: George Allen & Unwin.

1918a. *Mysticism and Logic and Other Essays*. London: Allen & Unwin.

1918b. "The Philosophy of Logical Atomism. Lectures I–II". *The Monist* 28:495–527.

1919a. *Introduction to Mathematical Philosophy*. London: George Allen & Unwin.

1919b. "The Philosophy of Logical Atomism. Lectures III and IV". *The Monist* 29 (1):32–63.

1927. *The Analysis of Matter*. London: Kegan Paul, Trench, Trubner & Co.

1959. *My Philosophical Development*. London: Allen & Unwin.

1984. "Theory of Knowledge: The 1913 Manuscript". In *The Collected Papers of Bertrand Russell. Vol 7*, edited by E. R. Eames and K. Blackwell. London: Allen & Unwin.

1994. "Necessity and Possibility". In *Collected Papers of Bertrand Russell, Vol. 4, Foundations of Logic, 1903–05*, edited by A. Urquhart and A. C. Lewis. London: Routledge, 507–520.

Schlick, Moritz. 1930. "Die Wende Der Philosophie". *Erkenntnis* 1 (1):4–11.

Schopenhauer, Arthur. 2010. *The World as Will and Representation*. Edited by C. Janaway, A. Welchman and J. Norman. Cambridge: Cambridge University Press.

2019. *On the Basis of Morality*. Translated by E. F. J. Payne. Indianapolis: Hackett.

Shapiro, Stewart. 2000. *Thinking About Mathematics. The Philosophy of Mathematics*. Oxford: Oxford University Press.

Shieh, Sanford. 2014. "In What Way Does Logic Involve Necessity?". *Philosophical Topics* 42 (2):298–337.

2015. "How Rare Is Chairman Mao?". In *Dummett on Analytical Philosophy*, edited by B. Weiss. London: Macmillan, 84–121.

2019. *Necessity Lost: Modality and Logic in Early Analytic Philosophy, Vol. 1*. Oxford: Oxford University Press.

2021. "What Could Be the Great Debt to Frege? Or, Gottlobius Ab Paene Omni Naevo Vindicatus". *Disputatio. Philosophical Research Bulletin* 10 (18):5–62.

2022. "On Attempting to Solve the Direction Problem". *Russell: The Journal of the Bertrand Russell Research Centre* 42:132–168.

In press-a. *Necessity Regained: Modality and Logic in Early Analytic Philosophy, Vol. 2*. Oxford: Oxford University Press.

In press-b. *Wittgenstein and Russell, Elements of the Philosophy of Ludwig Wittgenstein*. Cambridge: Cambridge University Press.

Skolem, Thoralf. 1967. "The Foundations of Elementary Arithmetic Established by Means of the Recursive Mode of Thought, without Use of Apparent Variables Ranging over Infinite Domains". In *From Frege to Gödel. A Sourcebook in Mathematical Logic. 1879–1931*, edited by J. van Heijenoort. Cambridge, MA: Harvard University Press, 303–333.

Sölle, Dororthee. 1997. *Mystik Und Widerstand: "Du Stilles Geschrei"*. Hamburg: Hoffmann und Campe.

Somavilla, Ilse, ed. 1997. *Ludwig Wittgenstein, Denkbewegungen: Tagebucher 1930–1932, 1936–1937 (Ms 183)*. Innsbruck: Haymon.

ed. 2006. *Wittgenstein – Engelmann: Briefe, Begegnungen, Erinnerungen*. Innsbruck: Haymon.

Spinoza, Baruch. 1992. *Ethics. Treatise on the Emendation of the Intellect and Selected Letters*. Translated by S. Shirley. Edited by S. Feldman. Indianapolis: Hackett.

Stebbing, Susan. 1933. "Logical Positivism and Analysis". *Proceedings of the British Academy* 19:53–87.

Stegmüller, Wolfgang. 1966. "Eine Modelltheoretische Präzisierung Der Wittgensteinschen Bildtheorie". *Notre Dame Journal of Formal Logic* 7 (2):181–195.

Stein, Edith. 1959. *Die Frau. Ihre Aufgabe Nach Natur Und Gnade*. Freiburg: Herder.

Stenius, Erik. 1960. *Wittgenstein's Tractatus. A Critical Exposition of Its Main Lines of Thought*. Oxford: Basil Blackwell.

Sullivan, Peter. 1994. "The Sense of 'a Name of a Truth-Value'". *Philosophical Quarterly* 44:476–481.

1996. "The 'Truth' in Solipsism, and Wittgenstein's Rejection of the a Priori". *European Journal of Philosophy* 4 (2):195–219.

2011. "Synthesizing without Concepts". In *Beyond the Tractatus Wars: The New Wittgenstein Debate*, edited by R. Read and M. A. Lavery. New York and Abingdon: Routledge, 171–189.

2013. "Idealism in Wittgenstein: A Further Reply to Moore". In *Wittgenstein's Tractatus: History and Interpretation*, edited by P. Sullivan and M. Potter. Oxford: Oxford University Press, 256–270.

Sundholm, Göran. 1992. "The General Form of the Operation in Wittgenstein's *Tractatus*". *Grazer philosophische Studien* 42:57–76.

Sullivan, Peter, and Colin Johnston. 2018. "Judgments, Facts, and Propositions: Theories of Truth in Russell, Wittgenstein, and Ramsey". In *The Oxford Handbook of Truth*, edited by M. Glanzberg. Oxford: Oxford University Press, 150–192.

Tang, Hao. 2011. "Transcendental Idealism in Wittgenstein's Tractatus". *The Philosophical Quarterly* 61 (244):598–607.

Tejedor, Chon. 2015. *The Early Wittgenstein on Metaphysics, Natural Science, Language and Value*. London: Routledge.

Trybus, Adam. 2020. "Two of a Kind: Setting the Record Straight on Russell's Exchange with Ladd-Franklin on Solipsism". *Russell* 39:101–120.

Urquhart, Alasdair. 2003. "The Theory of Types". In *The Cambridge Companion to Bertrand Russell*, edited by N. Griffin. Cambridge: Cambridge University Press, 286–309.

van der Schaar, Maria. 2018. "Frege on Judgement and the Judging Agent". *Mind* 127:225–250.

forthcoming, "Russell on Judgment and the Judging Subject". In *Early Analytic Philosophy: Origins and Transformations*, edited by J. F. Conant and G. Nir. New York: Routledge.

von Wright, Georg Henrik. 1982. "Modal Logic and the *Tractatus*". In *Wittgenstein*. Oxford: Basil Blackwell, 185–200.

Waismann, Friedrich. 1965. "Notes on Talks with Wittgenstein". *The Philosophical Review* 74 (1):12–16.

1979. *Wittgenstein and the Vienna Circle*. Oxford: Blackwell.

1986. "The Nature of Mathematics: Wittgenstein's Standpoint". In *Ludwig Wittgenstein. Critical Assessments*, edited by S. Shanker. London: Croom Helm, 60–67.

Weiner, Joan. 2001. "Theory and Elucidation: The End of the Age of Innocence". In *Future Pasts: Perspectives on the Analytic Tradition in Twentieth Century Philosophy*, edited by J. Floyd and S. Shieh. New York: Oxford University Press, 43–66.

Weininger, Otto. 1903. *Geschlecht und Charakter. Eine Prinzipielle Untersuchung*. Vienna and Leipzig: Wilhelm Braumüller Verlag.

1906. *Sex and Character*. New York and Chicago, IL: A.L. Burt Company Publishers.

Weiss, Max. 2017. "Logic in the Tractatus I: Definability". *Review of Symbolic Logic* 10 (1):1–50.

White, Roger M. 2006. *Wittgenstein's Tractatus Logico-Philosophicus: Reader's Guide*. London and New York: Continuum.

Whitehead, Alfred North, and Bertrand Russell. 1910. *Principia Mathematica*. Cambridge: Cambridge University Press.

1925. *Principia Mathematica*. 2nd ed. Cambridge: Cambridge University Press.

Williams, Bernard. 1974. "Wittgenstein and Idealism". In *Understanding Wittgenstein, Royal Institute of Philosophy Lectures, Vol. 7, 1972–73*, edited by G. Vesey. London: Macmillan, 76–95.

Winch, Peter. 1972. *Ethics and Action*. London: Routledge & Kegan Paul.

Wittgenstein, Ludwig. 1921. "Logisch-Philosophische Abhandlung". In *Annalen der Naturphilosophie*, edited by W. Ostwald. Leipzig: Verlag Unesma, 185–262.

1922. *Tractatus Logico-Philosophicus*. Translated by C. K. Ogden. London: Routledge and Kegan Paul.

1929. "Some Remarks on Logical Form". *Aristotelian Society Supplementary Volume* 9:162–171.

1965. "A Lecture on Ethics". *The Philosophical Review* 74 (1):3–12.

1967. *Zettel*. Translated by G. E. M. Anscombe. Edited by G. E. M. Anscombe and G. H. von Wright. Berkeley: University of California Press.

1969a. *The Blue and Brown Books*. 2nd ed. Oxford: Basil Blackwell.

1969b. *Briefe an Ludwig Von Ficker*. Edited by G. H. von Wright and W. Methlagl. Salzburg: Otto Müller Verlag.

1971. *Prototractatus. An Early Version of Tractatus Logico-Philosophicus*. Translated by D. Pears and B. F. McGuinness. Edited by B. McGuinness, T. Nyberg and G. H. von Wright. London: Routledge & Kegan Paul.

1973. *Letters to C. K. Ogden with Comments on the English Translation of the Tractatus Logico-Philosophicus*. Oxford: Basil Blackwell.

1974a. *Philosophical Grammar*. Translated by A. Kenny. Edited by R. Rhees. Oxford: Blackwell.

1974b. *Tractatus Logico-Philosophicus*. Translated by D. F. Pears and B. McGuinness. 2nd ed. London: Routledge and Kegan Paul.

1975. *Philosophical Remarks*. Translated by R. Hargreaves and R. White. Edited by R. Rhees. Oxford: Blackwell.

1976. *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge, 1939, from the Notes of R. G. Bosanquet, Norman Malcolm, Rush Rhees, and Yorick Smythies*. Edited by C. Diamond. Ithaca, NY: Cornell University Press.

1979. *Notebooks, 1914–1916*. Translated by G. E. M. Anscombe. Edited by G. H. von Wright and G. E. M. Anscombe. 2nd ed. Oxford: Blackwell.

1989. *Logisch-Philosophische Abhandlung. Tractatus Logico-Philosophicus. Kritische Edition*. Edited by B. McGuinness and J. Schulte. Frankfurt am Main: Suhrkamp.

1993. *Ludwig Wittgenstein. Philosophical Occasions 1912–1951*. Edited by J. Klagge and A. Nordmann. Indianapolis and Cambridge: Hackett.

1999. *Culture and Value*. Translated by P. Winch. Edited by G. H. von Wright. 2nd ed. Oxford: Blackwell.

2000. *Wittgenstein's Nachlass: Text and Facsimile Version*. The Bergen Electronic Edition. Oxford: Oxford University Press.

2005. *The Big Typescript. Ts 213*. Translated by C. G. Luckhardt and M. A. E. Aue. Edited by C. G. Luckhardt and M. A. E. Aue. Oxford: Blackwell.

2009. *Philosophical Investigations*. Translated by G. E. M. Anscombe, P. M. S. Hacker and J. Schulte. Edited by P. M. S. Hacker and J. Schulte. 4th ed. Oxford: Blackwell.

2016. *Wittgenstein: Lectures, Cambridge 1930–1933 from the Notes of G. E. Moore*. Edited by D. Stern, B. Rogers and G. Citron. Cambridge: Cambridge University Press.

Zalabardo, José L. 2015. *Representation and Reality in Wittgenstein's Tractatus*. Oxford: Oxford University Press.

2018a. "Response to Commentaries on 'The *Tractatus* on Unity'". *Australasian Philosophical Review* 2:343–354.

2018b. "The *Tractatus* on Unity". *Australasian Philosophical Review* 2:250–271.