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Mechanics without mechanisms

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ABSTRACT

At the time of Heinrich Hertz's premature death in 1894, he was regarded as one of the leading scientists of his generation. However, the posthumous publication of his treatise in the foundations of physics, *Principles of Mechanics*, presents a curious historical situation. Although Hertz's book was widely praised and admired, it was also met with a general sense of dissatisfaction. Almost all of Hertz's contemporaries criticized *Principles* for the lack of any plausible way to construct a mechanism from the "hidden masses" that are particularly characteristic of Hertz's framework. This issue seemed especially glaring given the expectation that Hertz's work might lead to a model of the underlying workings of the ether.

In this paper I seek an explanation for why Hertz seemed so unperturbed by the difficulties of constructing such a mechanism. In arriving at this explanation, I explore how the development of Hertz's image-theory of representation framed the project of *Principles*. The image-theory brings with it an austere view of the "essential content" of mechanics, only requiring a kind of structural isomorphism between symbolic representations and target phenomena. I argue that bringing this into view makes clear why Hertz felt no need to work out the kinds of mechanisms that many of his readers looked for. Furthermore, I argue that a crucial role of Hertz's hypothesis of hidden masses has been widely overlooked. Far from acting as a proposal for the underlying structure of the ether, I show that Hertz's hypothesis *ruled out* knowledge of such underlying structure.

1. Introduction

On New Year's day of 1894, Heinrich Hertz died just 36 years old. He had been heralded as one of the most promising scientists of his generation—'Hertz seemed to be predestined to open up to mankind many of the secrets which nature has hitherto concealed from us', as Helmholtz put it (Hertz (1899), vii). Hertz had dedicated the last few years of his life to a grand project in the foundations of physics, culminating in the posthumous publication of *Principles of Mechanics*. As he had prepared to send the manuscript to press, Hertz expressed trepidation about how it would be received, revealing to his parents that he had never shown it to another soul.[1] When *Principles* finally appeared it was received with high praise, but even as it was admired for its elegance and scope Hertz's contemporaries could not find in it the kinds of advances that they had hoped for. Indeed, there was a general sense of confusion regarding what *Principles* was supposed to have achieved. Hertz himself, of course, could not help. As Boltzmann lamented, at the same moment that Hertz's book was published 'his lips became for ever sealed to the thousand requests for clarification that are certainly not on the tip of my tongue alone' (Boltzmann (1974), p. 90).

Nevertheless, *Principles* went on to have a remarkable impact on both physicists and philosophers. It has been said that Hertz's book marks 'the beginning of modern physics' (Mulligan (2001), p. 151), a view defended emphatically by Cassirer and echoed more recently by van Fraassen.[2] *Principles* was also 'cited by Wittgenstein (in the *Tractatus*) and by Carnap (in the *Aufbau*), and even where it was poorly received (by Poincaré and Duhem) its influence was strongly felt' (Saunders (1998), p. 123). Indeed, almost all the leading physicists and scientifically-oriented philosophers of two generations read and reacted to Hertz's book.[3] Crucially, however, almost all of these esteemed readers found Hertz's mechanics 'interesting and beautiful, but either baffling or unsuccessful, or both' (Preston (2008a), p. 100). The sweeping influence of *Principles* makes the problem of finding a satisfactory interpretation of it all the more

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[1] Cf. Hertz's letter to his parents, 19 November 1893 (Hertz (1977), p.343).

[2] Cf. Cassirer (1950), p. 114 ff., and van Fraassen (2008), p. 204 ff.

[3] Besides Poincaré and Duhem, Preston names Helmholtz, Mach, Boltzmann, Lorentz, FitzGerald, Einstein and Russell (Preston (2008a), p. 100).

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pressing, yet the difficulties in doing so remain as acute today as they did following Hertz's untimely death.

Principles begins with three primitive notions: space, time, and mass. It proceeds to build up a sophisticated analytical framework in which to treat the mechanical properties of "systems", defined as collections of material points with *connections* between them (equations relating their relative positions). Hertz then posits one fundamental law: 'Every free system persists in its state of rest or of uniform motion in a straightest line' (§309).[4] The grand claim of the book is that the entire factual content of classical mechanics is captured in this single statement. However, *Principles* does not merely aim to treat mechanics more economically and systematically than previous formulations. Hertz also intends to demystify the notions of *force* and *energy*, deriving cleaned up versions of both from the spatial and temporal relations between masses. Hertz claims that by avoiding obscurities in Newton's formulation of the laws of mechanics, certain confused questions which troubled his contemporaries simply won't arise.[5] To achieve all this, and to apply his framework to the full range of mechanical phenomena, Hertz introduces the notion of *hidden masses*:

> If we wish to obtain an image of the universe which shall be well-rounded, complete, and conformable to law, we have to presuppose, behind the things which we see, other, invisible things—to imagine confederates concealed beyond the limits of our senses … We are free to assume that this hidden something is nought else than motion and mass again, motion and mass which differ from the visible ones not in themselves but in relation to us and to our usual means of perception. (Hertz (1899), p.25)

However, it is here that we encounter the confused reaction of Hertz's readers. Helmholtz, in the introduction he wrote for *Principles*, remarked: 'Unfortunately [Hertz] has not given examples illustrating the manner in which he supposed such hypothetical mechanism to act; to explain even the simplest cases of physical forces on these lines will clearly require much scientific insight and imaginative power' (Hertz (1899) xx). Boltzmann went to considerable effort to try and construct the mechanisms that Hertz had apparently left out but without success, remarking: 'so long as even in the simplest cases no systems or only unduly complicated systems of hidden masses can be found that would solve the problem in the sense of Hertz's theory, the latter is only of purely academic interest' (Boltzmann (1974), p. 90). And Mach was particularly pointed in drawing attention to the fact that such Hertzian mechanisms would oblige one 'to resort, even in simplest cases, to fantastic and even frequently questionable fictions' (Mach (1960), p. 323). Modern commentators have been similarly unanimous in complaining about the difficulties of finding plausible Hertzian mechanisms. Lützen remarks, 'If Hertz had lived he would certainly have been hard pressed for a reaction to this problem' (Lützen (2005), p. 278), or as Mulligan puts it, 'This criticism is quite valid and undoubtedly carried great weight with physicists in the decade after 1894' (Mulligan (1998), p. 178).

The central goal of this paper will be to resolve this persistent tension in interpreting Hertz's book. I will begin by situating *Principles* in its historical context, and identify the widespread tendency to regard Hertz's project as closely connected with the search for an ether mechanism. I will argue that this tendency has contributed to the confusion and dissatisfaction amongst Hertz's readers because it ties the value of his project to the prospects of finding such a mechanism. I will then turn to discuss Hertz's ideas concerning scientific representation. These ideas culminated in the "image-theory" expressed in the long introduction to *Principles*, in which Hertz presented a strikingly austere account of the essential content of a scientific theory. Applying this in interpreting the book as a whole, I will argue that it has been misleading to interpret *Principles* as closely connected with the quest for an ether mechanism, despite passages where Hertz himself seems to invite such an interpretation. More specifically, I will argue that a crucial role of Hertz's hypothesis of hidden masses has been widely overlooked. Rather than acting as a proposal for the underlying structure of the ether, Hertz's hypothesis *rules out* knowledge of "fundamental" underlying structure.

This paper thus aims to bring together two strands within the Hertz literature. On the one hand, scientists and historians who have been impressed with the scope and elegance of Hertz's book have not been able to account for his seemingly cavalier attitude towards the difficulties of constructing mechanisms of hidden masses. On the other hand, philosophers who have been intrigued by the subtle account of scientific representation in Hertz's introduction have made few attempts to interpret the book as a whole. I will endeavour to show that the philosophical morals of Hertz's introduction hold the key to dispelling the confusion that has surrounded *Principles* ever since it was published.

2. The quest for an ether mechanism

The second half of the nineteenth century that encompassed Hertz's short career was characterized by fervent research in electromagnetism. The first volume of Maxwell's *Treatise* appeared in 1873, and Hertz's own groundbreaking observations of electric waves in 1888 established Maxwell's theory as canonical. For many physicists the most appealing aspect of that theory was the way in which it seemed to eschew instantaneous actions-at-a-distance in favour of the notion of waves propagating through a medium. Hertz's famous experiments were widely regarded as confirming this view of electromagnetism, and Kelvin introduced Hertz's collection of papers on the subject as a 'splendid consummation' of 'the nineteenth-century school of plenum, one ether for light, heat, electricity, magnetism' (Hertz (1893) xv).[6] However, finding an ether mechanism which could account for electromagnetic phenomena remained a critical open problem.

In seeking an ether mechanism many of Hertz's contemporaries were inspired by the success of the kinetic theory of gases. That conception of a gas—a swarm of billiard-ball like atoms, colliding with each other according to ordinary Newtonian mechanics—had been extremely successful in both accounting for thermodynamical properties and leading to novel predictions. It was also admired for conveying a particularly satisfactory kind of understanding: the model *really represented* what a gas was like, at least approximately. Hence a widely held view was that it 'ought to be possible, at least in principle, to do the same thing for the ether: to find a mechanical model that reflected its true nature' (Hunt (1991), pp. 76–77).

Thus the historical context in which *Principles* appeared involved a plethora of increasingly intricate attempts to show how some kind of material ether, governed by ordinary mechanics, could account for electromagnetic effects. More generally, the promise of an ether mechanism that eschewed action-at-a-

[4] For the remainder of this paper, a section number without a further citation will be used to refer to passages from the main body of *Principles*. Note that I have made occasional modifications to the published translation.

[5] Cf. Hertz (1899), p. 8.

[6] See also Mulligan (2001), p. 143: 'Hertz empirically confirmed Maxwell's electromagnetic waves; it was universally assumed that the ether was confirmed at the same time'.

distance was a defining feature of theoretical physics around 1890, and the background against which Hertz turned to foundational work in mechanics. Indeed, an eloquent description of this situation is due to Hertz himself:

> More and more we feel that [the nature of the ether] is the all-important problem, and that the solution of it will not only reveal to us the nature of what used to be called imponderables, but also the nature of matter itself and of its most essential properties – weight and inertia. The quintessence of ancient systems of physical science is preserved for us in the assertion that all things have been fashioned out of fire and water. Just at present physics is more inclined to ask whether all things have not been fashioned out of the ether. (Hertz (1896), pp.326 f.)

However, proposing a concrete ether mechanism was clearly not a *direct* goal of Hertz's book. In fact, before *Principles* was published Hertz had explicitly attempted to dispel such rumours concerning what it was he was working on:

> What you have heard about my works … is unfortunately without any foundation and I do not know how this opinion has been formed. I have not at all worked with the mechanics of the electric field, and I have not obtained anything concerning the motion of the ether. (Hertz to Emil Cohn, November 25, 1891)[7]

Hertz's primary aim, as he himself emphasised, was to achieve a certain kind of *clarification* of classical mechanics as it stood.[8] Thus most readers of *Principles*—both historical and contemporary—have regarded it as an attempt to lay the groundwork for some future ether mechanism, the details of which could be filled in later. But the inclination towards interpreting *Principles* this way has contributed to the dissatisfaction amongst Hertz's readers, for it ties the value of his project to the prospects of filling in these details. We thus encounter a crucial unanswered question: How could Hertz's apparent attitude towards the difficulties of constructing such a mechanism have been so cavalier? Indeed, independently of the historical context, the content of *Principles* can also seem to invite this question itself.

3. An overview of *principles*

In this section it is important to note that I move brusquely over much fine detail that warrants further discussion.[9] Before proceeding, a note on terminology. In the opening paragraphs of *Principles* we find Hertz introducing 'material particles' and 'material points' in an interconnected series of definitions, leading up to the definition of a 'system'. The latter terms are literal translations from the German (*materieller Punkte* and *System* respectively), but translating Hertz's term *Massenteilchen* as 'material particle' is misleading. Hertz's *Massenteilchen* are, in an important sense, smaller—indeed, infinitely smaller—than his material points, and this conflicts with the ordinary understanding of 'particles' and 'points' in English. To avoid unhelpful associations, I will use '*Massenteilchen*' instead of 'material particle(s)' in what follows.[10]

3.1. Hertz's analytical framework

Principles is divided into two books: in the first, Hertz defines his terms and establishes an analytical (mathematical) framework; in the second, he explains how this framework is to be applied. The first book purports to be a priori 'in Kant's sense':

> The subject-matter of the first book is completely independent of experience. All the assertions made are a priori judgments in Kant's sense. They are based upon the laws of the internal intuition of, and upon the logical forms followed by, the person who makes the assertions; with his external experience they have no other connection than these intuitions and forms may have. (§1)[11]

The content of the first book is supposed to be compatible with any interactions with spatio-temporal objects whatsoever. Thus it is only in book two that we find the one proposition that Hertz regards as falsifiable: his fundamental law.

Following Kant, Hertz helps himself to 'the space of Euclid's geometry' and 'the time of our internal intuition' (§2). In the case of mass, however, there is no associated Kantian form of intuition to appeal to, and Hertz's avoidance of anything dependent on experience leads to a very minimal notion: the "mass" contained in a given space is defined as the relative number of *Massenteilchen* in that space. Hence Hertz first defines *Massenteilchen* in order to give his definitions of mass, then proceeds to definitions of material points and, finally, systems.

Massenteilchen are represented completely by functions of temporal coordinates, associating spatial locations with moments of time:

Definition 1. A Massenteilchen is a characteristic by which we associate without ambiguity a given point in space at a given time with a given point in space at any other time. (§3)

Hertz also stipulates that any number of *Massenteilchen* can occupy the same location at the same time, allowing for the two definitions that follow:

Definition 2. The number of Massenteilchen in any space, compared with the number of Massenteilchen in some chosen space at a fixed time, is called the mass contained in the first space.

We may and shall consider the number of *Massenteilchen* in the space chosen for comparison to be infinitely great. The mass of the separate *Massenteilchen* will therefore, by the definition, be infinitely small. The mass in any given space may therefore have any rational or irrational value. (§4)

Definition 3. A finite or infinitely small mass, conceived as being contained in an infinitely small space, is called a material point. (§5)

A material point may at first seem to be the familiar "point mass" by which standard presentations of mechanics routinely treat stars and atoms alike: a discrete object whose mass can be treated as situated at a point. However, according to Hertz's definition of mass it must be possible for material points to contain infinite numbers of *Massenteilchen* if their mass values are to range over the real numbers. Hertz claims we can do this by 'supposing the *Massenteilchen* to be of a higher order of infinitesimals than those material

[7] Translations of the letter are reproduced in Lützen (2005), p. 74 and Nordmann (1998), p. 160. For an extract of the original German text see Nordmann (1998), p. 169.

[8] The letter to Cohn continues: 'This summer I have thought a great deal about the usual mechanics … In this area I would like to put something straight and arrange the concepts in such a way that one can see more clearly what are the definitions and what are the facts of experience, such as, for example, concepts of force and inertia. I am already convinced that it is possible to obtain great simplifications here'.

[9] As a starting place, Lützen's (2005) book-length commentary on *Principles* is an extremely valuable resource for a discussion of the details of Hertz's work.

[10] In this I follow Lützen (2005), cf. p.135.

[11] For some discussion of Hertz's Kantian influences, see Hyder (2002), pp. 35–46, Lützen (2005) §10, and Leroux (2001), p. 192 f.

points which are regarded as being of infinitely small mass' (§5).[12] This relationship between the material points and the *Massenteilchen* is suggestive of the material points in continuum mechancis, which are integrated over to define the properties of continuous media. In fact, Hertz's introduction of *Massenteilchen* might have been intended, in part, as a way to preserve conservation of mass whilst allowing for continually varying mass-densities.[13]

The final definition in the first chapter is of a *system*:

Definition 4. A number of material points considered simultaneously is called a system of material points, or briefly a system. The sum of the masses of the separate points is, by §4, the mass of the system. (§6)

Systems are simply collections of point masses, 'considered simultaneously'. For the remainder of *Principles* Hertz concerns himself entirely with the mechanics of material systems (cf. §121), and shows that the connections of such a system can always be represented by 'equations of condition' of a canonical form (cf. §115 ff.). A great part of the ensuing work is in setting up the vocabulary to talk about the properties of such systems (their displacement, velocity, acceleration, and so on), and this vocabulary finds a natural home in the context of the *configuration space* associated with a system, to which we can now turn.[14]

3.2. Configuration space

The basic idea of a system's configuration space is straightforward. A system of n material points has an associated configuration space with $3n$ dimensions—one dimension for each of the three coordinates of each of its points—so that every location in configuration space represents a conceivable position of the whole system. For example, the position of a system of three points can be given by specifying the nine coordinates in its associated configuration space.

When there are connections between the points there are corresponding limitations on which regions of configuration space are accessible. Specifically, each connection rules out the region that would correspond to "breaking" that connection. A rigid system in which no material point can move independently of any of the others has only six degrees of freedom; hence, no matter how many material points it has, such a system will always be located within a 6-dimensional subspace inside its configuration space. In general, the connections of a system always limit the accessible region of a $3n$-dimensional configuration space to a lower-dimensional subspace.[15]

Many of the key geometric properties of configuration space are given with its metrical properties, which Hertz derives by first defining the 'magnitude of the displacement of a system':

> The magnitude of the displacement of a system is the quadratic mean value of [i.e. the positive root of the arithmetic mean of the squares of] the magnitudes of the displacements of all its *Massenteilchen*. (§§28, 29)

Note here the reference to *Massenteilchen*.[16] If Hertz had calculated the displacements of the material points this would have resulted in configuration space having a standard Euclidean metric. In other words, the line element of configuration space would have taken the familiar "Pythagorean" form:

$$ds^2 = \sum_{i=1}^{3n} dx_i^2$$

However, calculating the displacements of the *Massenteilchen* instead of the material points "weights" the expression for the magnitude of the displacement of a system, so that the more massive points contribute more to the displacement. Hertz thus has the raw material to develop a more exotic metric for configuration space, first moving to a definition of *infinitesimal* displacement of a system (cf. §54), and then to expressions for the lengths and curvatures of paths of systems in general (cf. §§104 ff.). This results in the line element of configuration space having the following form:

$$ds^2 = \sum_{i=1}^{3n} m_i dx_i^2$$

Weighting the expression for (infinitesimal) displacement thus links the metrical properties of configuration space to the particular mass distribution of the system at hand. To appreciate the significance of this metric structure, it is helpful to approach it from a different direction.[17] If the velocity of the i-th material point is v_i, the total kinetic energy of the system is given by:

$$T = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2$$

From here, we could define the line element of configuration space as follows:

$$ds^2 = 2T dt^2 = \sum_{i=1}^{n} m_i v_i^2 dt^2$$

As $v_i = (dx_i^2 + dy_i^2 + dz_i^2)^{\frac{1}{2}}/dt$ this gives:

$$ds^2 = \sum_{i=1}^{n} m_i \left(dx_i^2 + dy_i^2 + dz_i^2\right)$$

Denoting the coordinates of the μ-th point as $(x_{3\mu-2}, x_{3\mu-1}, x_{3\mu})$, and letting its mass be equal to $m_{3\mu-2} + m_{3\mu-1} + m_{3\mu}$, we can see that we have recovered Hertz's expression for the line element:

$$ds^2 = \sum_{i=1}^{3n} m_i dx_i^2$$

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[12] Although Hertz is fudging the mathematical details here, we could flesh this out on Hertz's behalf using modern tools. For one suggestion along these lines see Lützen (2005), p. 139.

[13] For a discussion of attempts that were made to extend Hertz's framework to continuous systems, see Lützen (2005), p. 140 and p.286. Note that, because Hertz's mechanics seems only directly applicable to discrete systems, commentators have not drawn on concepts in continuum mechanics in interpreting either Hertz's *Massenteilchen* or his material points. Although this may be a mistake, a full discussion of this issue is beyond the scope of this paper. I hope to examine these matters in future work.

[14] Hertz himself minimized his use of spatial language in this context, and in particular did not use the term 'configuration space'. This is because Hertz was keen to play down any direct comparison between mathematical high-dimensional spaces and *physical* space. For a brief discussion of this point, see Lützen (2005), p. 110.

[15] In fact this is only true for *holonomous* connections (cf. §123). Hertz regarded it as important to incorporate non-holonomous connections within his framework, even though he could have regarded these as ultimately derivable from holonomic connections—cf. Lützen (2005), p. 193. In this section and the following I mainly limit my attention to holonomous systems; for some discussion of non-holonomous systems see Lützen (2005) §15.3.

[16] The need for the appearance of *Massenteilchen* in this definition is in fact a key reason why Hertz included them in his framework at all. For a detailed discussion of the development of the idea of *Massenteilchen* in the early drafts of *Principles* see Lützen (2005), pp. 146–158.

[17] Here I follow Lanczos (1962), p. 22.

Hence the total kinetic energy of the system can be written as $T = \frac{1}{2} m \left(\frac{ds}{dt}\right)^2$, where m is the sum of the masses of the individual points. This means that the total kinetic energy of the system can be regarded as *the kinetic energy of a single point in configuration space with unit mass*. Situating a mechanical problem within a configuration space of this structure thus carries over the mechanics of a single point to the mechanics of an arbitrary system.[18]

If a system has no connections at all between its points it moves in a *straight* path in its configuration space (which is indeed the *straightest* path available). Increasingly complex systems will have an increasing number of connections between their points. As each connection defines a $(3n-1)$-dimensional hypersurface inside the system's configuration space, and as the path that a system traces out must lie on the intersection of the hypersurfaces determined by all of its connections, every additional connection causes the system's path to deviate further from the straight path that it would otherwise follow. Thus every new connection increases the curvature of the system's path. Hertz's fundamental law asserts that the motion of a free material system (roughly, one that can be treated as isolated) always traces out a straightest path on this curved hypersurface, embedded within its $3n$-dimensional configuration space.[19]

The full elegance of Hertz's fundamental law as a kind of generalization of the principle of inertia is thus revealed. In Hertz's words: '[the fundamental law] asserts that if the connexions of the system could be momentarily destroyed, its masses would become dispersed, moving in straight lines with uniform velocity, but that as this is impossible, they tend as nearly as possible to such a motion' (Hertz (1899), p. 28).

3.3. Hidden masses and cyclical coordinates

From what has been said so far it remains opaque how Hertz's fundamental law, on its own, could accommodate all the varied phenomena of mechanics. Of course, many canonical mechanical problems concern systems that are *not* free, such as systems acted on by forces. To capture such systems within the scope of his fundamental law, Hertz allows a 'complete' free system to be decomposed into subsystems, and, in particular, to contain a *hidden* subsystem (cf. §429). Thus Hertz introduces the 'hidden masses' that are particularly characteristic of his framework. This idea plays a fundamental role for Hertz: as already noted, it is what allows him to employ only space, time and mass as his primitive notions, and gives rise to one of the key advantages that he believes his own formulation of mechanics has over other formulations. For although Hertz thinks that the attempt to unify phenomena in a law-like way inevitably requires stipulating something that is not directly observable, he makes the case that this does not necessitate an appeal to a further primitive notion: 'We may admit that there is a hidden something at work, and yet deny that this something belongs to a special category.' (Hertz (1899), p. 25).[20]

Hertz goes on, 'What we are accustomed to denote as force and as energy now become nothing more than an action of mass and motion, but not necessarily of mass and motion recognisable by our coarse senses.' (Hertz (1899), p. 26). Here, Hertz appeals to Helmholtz's earlier work on cyclical systems. A cyclical coordinate is one whose effect on the properties of a system is due only to its change, not its absolute value. A system is then called cyclical if its energy can be approximated as a function of the rates of change of its cyclical coordinates (cf. §§546–549). As an intuitive example, consider the spinning ring of a gyroscope.[21] Each component part of the ring is immediately replaced by its neighbour as the gyroscope rotates. The positions of these components are thus paradigm cyclical coordinates: it is only their *rates of change* that affect the gyroscope's behaviour. Because of the conservation of angular momentum, a closed box with a spinning gyroscope fixed to the inside will resist certain changes in its motion, and hence such a setup could mimic the actions of an external force field.

The mathematical tools for describing hidden cyclical subsystems can thus be used to widen the scope of Hertz's fundamental law, accounting for motions which would ordinarily be explained by appealing to distant forces. In the general case Hertz treats a material system acted on by forces as *coupled* to one or more other (hidden) material systems, such that the systems have at least one coordinate in common (§450). He then defines a force as the effect that one such coupled system has upon the motion of another (§455). Hertz goes on to show that defining force in this way aligns with the notion of force in customary approaches to mechanical problems to a remarkable degree. However, Hertz's notion of force adds nothing beyond the application of the fundamental law to a system of connected material points—the complete system formed by the coupled systems is itself free and hence moves on a straightest path in its own configuration space.

Thus, after deriving all the canonical treatments[22] of mechanical problems within his analytical framework, Hertz claims that *Principles* is 'capable of embracing the whole content of ordinary mechanics' (Hertz (1899) xxii), and that 'no definite phenomena can at present be mentioned which would be inconsistent with the system' (Hertz (1899), p. 36).

4. Hertz and ether mechanisms

At this point we can take a step back and consider the basis for the general inclination to regard Hertz as concerned with laying foundations for an ether mechanism. Hertz's *Massenteilchen* can seem to be fundamental particles of some kind, and he proposes that hidden cyclical subsystems can model the effects of distant forces. His project can thus seem to bear a close relationship with certain nineteenth century attempts to model the ether. A particularly noteworthy example is the "gyrostatic adynamic" ether mechanism proposed by Kelvin a few years before *Principles* was published (cf. Schaffner (1972), pp. 194–203). In introducing this mechanism, Kelvin began by describing a network of spherical atoms arranged such that each lies at the centre of a tetrahedron of four others, linked to its four neighbours by rigid bars. The bars attach to the atoms in such a way that their end points can slide freely on the atoms' surfaces, allowing the whole structure to have a degree of flexibility. Furthermore, each bar is

[18] Cf. Lanczos (1962), p. 22: 'In this space one point is sufficient to represent the mechanical system, and hence we carry over the mechanics of a free particle to any mechanical system if we place that particle in a space of the proper number of dimensions and proper geometry'.

[19] A system can also be described in terms of its *general* coordinates (cf. §13) which directly characterize it in terms of its r degrees of freedom. Doing so still preserves the elegance of the fundamental law, but the motion of a free system will now trace out a straightest path in an associated r-dimensional Riemannian space. Importantly, the embedding $3n$-dimensional Euclidean space then disappears from view.

[20] As Nordmann notes, Hertz's approach in this regard has an eminently respectable pedigree, 'which can be traced back to Descartes and beyond' (Nordmann (1998), p. 169).

[21] Here I follow Wilson (2007), pp. 12–13.

[22] Including those of Lagrange, Hamilton, d'Alembert, Gauss and Jacobi, as well as Galileo and Newton—cf. Hertz (1899) Book 2 Chapter III.

conceived as containing, along its length, two miniature gyroscopes:

> Instead of a simple bar, let us take a bar of which the central part, for a third of its length for example, is composed of two rings in planes perpendicular to one another … Let the two rings be the exterior rings of gyroscopes, and let the axes of the interior rings be mounted perpendicularly to the line of the bar. (Schaffner (1972), p.195)[23]

Aligning the gyroscopes and setting them in motion gives the structure a kind of rotationally-dependent elasticity, differing from the behaviour of ordinary elastic solids due to the fact that the restoring forces depend on the rotations of the connecting bars away from their original orientations. Kelvin declared: ‘This relation of the quasi-elastic forces with rotation, is just that which we require for the ether, and especially to explain the phenomena of electro-dynamics and magnetism’ (Schaffner (1972), p. 196). Kelvin then used this structure as the basis for a significantly more intricate mechanism, designed to produce no restoring forces other than restoring couples in the same axes as deforming rotations.

Kelvin and Hertz can thus seem to have been closely aligned. Under such an interpretation, Hertz was clarifying mechanics with the expectation that a mechanism like Kelvin's would prove to be a good representation (or at least a useful analogy) of the structure of the ether. Importantly, we can see this style of interpretation directly informing the attempts that were made to fill in what appeared as the gaps in Hertz's presentation.[24] These attempts aimed to give Hertz's mechanics some plausibility by showing that it was at least *possible* to construct “Hertzian mechanisms”, crude and complicated as they might be.

Furthermore, there are certain passages in *Principles* which seem to suggest that Hertz was indeed hoping for precisely the kind of ether mechanism that many of his contemporaries were struggling to construct. The most overt such passage comes at the end of the introduction, where Hertz considers the merits of appealing to connections over distant forces, remarking: ‘the balance of evidence will be entirely in favour of the [Hertzian formulation] when a second approximation to the truth can be attained by tracing back the supposed actions-at-a-distance to motions in an all-pervading medium whose smallest parts are subjected to rigid connections’ (Hertz (1899), p. 41). Combining this with two other passages in which Hertz talks of “seeking the ultimate connections in the world of atoms” (to be discussed below, in section 6), it is hardly surprising that there exists an almost universal inclination to read *Principles* as aiming to provide foundations for an ether mechanism. At any rate, commentators such as FitzGerald felt no hesitation in interpreting Hertz this way:

> Hertz sees in all actions the working of an underlying structure whose masses and motions are producing the effects on matter that we perceive, and what we call force and energy are due to the actions of these invisible structures, which he implicitly identifies with the ether. (Hertz and Mulligan (1994), p.371)

Moreover, as we have seen, many modern commentators continue to interpret *Principles* along the same lines:

> [Hertz's] overwhelming conviction of the importance of the aether, joined to his urge to reduce all physics to mechanics, eventually culminated in 1894 in the posthumous publication of his *Mechanics*. (Mulligan (2001), p.138)[25]

Such interpretations make Hertz's apparent attitude towards the difficulties of constructing such a mechanism seem remarkably cavalier. Indeed, it is against this backdrop that the problem of finding a plausible Hertzian mechanism seems acutely pressing. However, this way of reading *Principles* doesn't fully take into account a crucial aspect of Hertz's book: the image-theory of scientific representation expressed in the introduction.

5. Hertz's image-theory

Commentators who have engaged closely with the philosophical content of Hertz's introduction have recognized Hertz as a progenitor of the family of “structuralist” views developed by figures in the philosophy of science throughout the twentieth century. Roughly speaking, such views regard the representative content of a scientific theory as stemming from its structural features rather than from the objects that it posits. Ernst Cassirer was perhaps the earliest commentator to recognize the importance of Hertz's role in this regard. Far from seeing *Principles* as laying foundations for an ether mechanism, Cassirer regarded Hertz's project as a response to the *problems* that had emerged in such attempts:

> Every barely imaginable suggestion and combination had been exhausted in an effort to establish [the ether's] constitution until finally, after all endeavors had failed, a change in the whole intellectual orientation was effected and investigators began to submit to critical proof the assumption of its existence instead of continuing to examine into its nature. (Cassirer (1950), p.89, cf. pp.103 ff.)

More recently, Leroux (2001) and van Fraassen (2008) have also emphasised Hertz's role in the movement away from the “mechanistic” approach encapsulated in the increasingly intricate nineteenth century attempts to find an ether mechanism. Van Fraassen even goes so far as to say, ‘In Hertz's, and later Poincaré's, verdict we recognize a definite *goodbye* to the interrelation of matter and ether as a live topic in physics’ (van Fraassen (2008), p. 202).

In seeking to understand the lack of mechanisms in Hertz's book, we need to appreciate how Hertz's ideas concerning scientific representation framed his project—in brief, how his philosophy framed his physics. Although the presentation of the “image-theory” in the introduction to *Principles* has been relatively well-

[23] For some discussion of Kelvin's model, see Schaffner (1972), pp. 68–75 and Stein (1981), p. 319.

[24] For brief surveys of these attempts, see Lützen (2005), p. 274 ff. and Preston (2008b), p. 59 ff.

[25] See also Saunders (1998), p. 126: ‘my own view of the *Principles* is that Hertz intended to make a methodological proposal, and that he supposed that it would be given substance by a mechanical model of ether’; and Lützen (2005), p. 266: ‘The sole aim of the book was to establish the theoretical foundation for a construction of such hidden systems or in other words for constructing a model of the ether’. Some commentators have even mistakenly claimed that *Principles* aimed to provide a *direct* model of the ether, cf. Hyder (2002), p. 42 f.: ‘the gap in Hertz's picture of electromagnetism was occupied by the ether: How are we to imagine its polarisation? … To fill the gap would need a picture of these hidden material systems. Hertz's last book, *The Principles of Mechanics Presented in a New Form*, attempted to do just this.’ However, other commentators have resisted the suggestion that the goal of *Principles* was to lay the groundwork for an ether mechanism. In particular, Nordmann has pointed ou that as Hertz's hidden masses are unobservable in principle, they are ‘not subject to exploration even by physical undertakings of the future’ (Nordmann (1998), p. 160). Hence Nordmann suggests that Hertz's primary focus revolved ‘around the conceptual problems of ordinary classical mechanics’ (ibid). In a similar vein, D'Agostino has remarked: ‘Since hidden quantities cannot be observed, they belong to a pure theoretical framework’(D'Agostino (1993), p. 73). I pursue a similar line of interpretation in section 6 below.

discussed in the literature,[26] it has not often been situated against the development of Hertz's earlier ideas.[27] Hertz discussed the role of "images" in scientific representation at least as early as his 1884 Kiel lectures[28]—a decade before *Principles* was published—and these ideas continued to develop throughout his work on electromagnetism.

The Kiel lectures are important for contextualizing Hertz's image-theory because it is here that Hertz introduced the distinction between the *essential* and *inessential* content of a scientific theory. Early in the lectures, Hertz discussed the desirability of gaining an image ('*Bild*') of the workings of nature without thereby ascribing to the phenomena any superfluous features that attach to the image via the imagination. An example where the imagination could be misleading would be attributing a colour to an atom simply because we can't imagine it otherwise. In such a case, Hertz argues, we simply have to regard colour as an *inessential* property, hence explicitly discount it as representing, or corresponding to, a property of the atom itself. Eight years later, having worked hard to distill the essential content out of Maxwell's sprawling *Treatise*, Hertz famously remarked:

> To the question, "What is Maxwell's theory?" I know of no shorter or more definite answer than the following: Maxwell's theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory. (Hertz (1893), p.21)

This is particularly important for our purposes for the following reason. In drawing attention to the difficulty of finding plausible mechanisms within the framework of *Principles*, both Helmholtz and Mach claimed that, in their own cases, they would remain content with the analytical representation given by the relevant systems of equations.[29] But the fact that Helmholtz and Mach both regarded themselves as thereby marking a contrast with Hertz is peculiar inasmuch as Hertz's concerns also lay precisely in the 'essential' content conveyed by the relevant equations, and had done so in a consistent and sustained way for a long time prior to his work on mechanics. In the context of Hertz's derivation of Maxwell's equations this is made particularly clear, as Hertz went on to say the following:

> If we wish to lend more colour to the theory, there is nothing to prevent us from supplementing all this and aiding our powers of imagination by concrete representations of the various conceptions … But scientific accuracy requires of us that we should in no wise confuse the simple and homely figure, as it is presented to us by nature, with the gay garment which we use to clothe it. (Hertz (1893), p.28)

We find here, again, the distinction between essential and inessential features that Hertz introduced in the Kiel lectures. In this context the 'simple and homely figure' presented by nature is the system of relations determined by Maxwell's equations, to which a 'gay garment' can be added, if desired, from amongst the competing hypotheses *about the underlying workings of an ether*. More generally, Hertz's proposal is that when we think carefully about the image of nature that a scientific theory conveys, we should attend to the *essential* features conveyed by that theory in its naked form. To do so, the theory 'should be so constructed as to allow its logical foundations to be easily recognized; all unessential ideas should be removed from it, and the relations of the essential ideas should be reduced to their simplest form' (Hertz (1893), p. 195). This is exactly the *clarification* that Hertz achieved with electromagnetism before he turned to classical mechanics.

5.1. The image-theory in principles

Hertz employed his image-theory in framing the entire purpose of *Principles*, and also in taking a stance from which to evaluate its success. With regard to the purpose of his book, Hertz was helpfully explicit in articulating his overall goal:

> The problem, whose solution the following investigation seeks, is this: to fill up the existing holes and specify a complete and definite presentation of the laws of mechanics, which is compatible with our present day knowledge, and in relation to the range of this knowledge is neither too narrow nor too broad. (Hertz (1899) xxi)

To understand the motivation to formulate a 'complete and definite presentation of the laws of mechanics', we need to consider Hertz's dissatisfaction with the already existing presentations. The development of the desiderata of a satisfactory presentation, and the comparison of the extant formulations of mechanics with Hertz's own novel reformulation on this basis, is the main task of his introduction. Hertz thus compares three competing formulations of mechanics: the traditional "Newtonian" formulation; the more recent "energetic" formulation (which attempted to derive the notion of force from the notion of energy); and Hertz's own formulation.

This is the context in which Hertz presents his image-theory. However, before narrowing his focus to scientific theories (and formulations of mechanics in particular), Hertz discusses how such "images" function in representation quite generally, beginning with the following:

> The procedure which we use in order to draw deductions of the future from the past, and thereby obtain the striven for foresight, is this: we make for ourselves inner simulacra [*Scheinbilder*] or symbols of external objects, and indeed we make them in such a way that the necessary consequences of the images [*Bilder*] in thought are always again the images of the necessary consequences of the pictured objects … The images of which we speak are our conceptions of things; they have with the things *one* essential conformity, which lies in the fulfillment of the aforementioned requirement. (Hertz (1899), p.1)

Hertz goes on to specify three criteria by which to evaluate images: *permissibility* (*Zuläßigkeit*), *correctness* (*Richtigkeit*), and *appropriateness* (*Zweckmäßigkeit*). In brief: Hertz's notion of permissibility can be glossed as the demand of logical consistency. The second criterion, of correctness, is given by Hertz's fundamental requirement on images: 'the necessary consequences of the images in thought are always again the images of the necessary consequences of the pictured objects in nature'. Thus the necessary consequents of a correct image give *successful predictions* of the relevant phenomena. Importantly, Hertz emphasizes that respecting the fundamental requirement is the *only* 'essential

[26] For example, Schaffner (1970), D'Agostino (1993), Majer (1998), and Lützen (2005) §§7–9.

[27] A notable exception is Lützen (2005), see in particular §8. See also van Fraassen (2008) §8, especially pp.201 ff.

[28] The lectures have been published in German, "Die Constitution der Materie" (Hertz, 2013). Although much of this material has not yet been studied in proper detail, for some initial discussion see Hyder (2002), pp. 35–46 and Lützen (2005), pp. 97–101.

[29] Cf. Hertz (1899) xix-xx, and Mach (1960), p. 321.

conformity' between the images and the objects. The final criterion, of appropriateness, is the most subtle of the three. Hertz distinguishes two separate strands which speak to the appropriateness of an image—its *distinctness* and its *simplicity*:

> Given two images of the same object, the more appropriate of them is the one which reflects more of the essential relations of the object than the other; the one which, we would say, is more distinct. Of two equally distinct images the more appropriate is the one which, besides the essential traits, contains the least number of unnecessary or empty relations, which is thus the simpler of the two. (Hertz (1899), p.2)

Hence it is incorporated into the criterion of appropriateness that we find a development of Hertz's distinction between essential and inessential features of an image. According to the account in *Principles*, one image is more distinct than another if it captures more of the relevant essential features. An image can further improve its appropriateness by being stripped of any inessential features. Such a 'naked' image is thereby simpler.

Note that everything so far is meant to apply to images understood very broadly as 'our conceptions of things'. It is only after he has specified the three criteria of permissibility, correctness and appropriateness that Hertz turns to consider the images provided by scientific theories. The key difference in the case of a scientific image is that it must be made clear which elements of the image are operative in meeting the different criteria, for only in this way is the systematic improvement of images possible. Nevertheless, as we shall see, there are important ways in which Hertz's three criteria are intimately connected. This becomes apparent if we examine the way Hertz employed the criteria of the image-theory in criticizing the traditional formulation of mechanics, thereby indicating what he thought stood to be gained through his own reformulation.

5.2. What principles achieved

Hertz set his three criteria to work in diagnosing what is problematic in 'the representation, differing in details but at root the same, in nearly every textbook which deals with the whole of mechanics, and in nearly every lecture course which disseminates the cumulative content of this science' (Hertz (1899), p. 4). In an important series of passages, Hertz presented several reasons to doubt the logical clarity of the traditional formulation of mechanics. He began with a critique of the notion of centrifugal force before turning to three 'general observations' as further evidence for his misgivings: the difficulty of expounding a rigorous and clear introduction to mechanics, the existence of disputes over the rigour of certain 'elementary' theorems, and the pervasiveness of questions concerning the nature of *force*. Hertz summarized the purpose of this extended polemic as follows:

> I have so severely questioned the permissibility of the image under consideration in these remarks that it must appear that it was my aim to dispute and eventually to deny its permissibility. But my aim, and my opinion, do not go so far as this. Such logical uncertainties, which make us anxious about the reliability of the foundations of the subject, though they really exist, have clearly not prevented a single one of the countless successes which mechanics has won in its application to the facts. Thus they could not stem from contradictions between the essential characteristics of our image, hence not from contradictions between those relations of mechanics which correspond to relations of things. Rather, they must be restricted to the inessential traits, to all those aspects which we ourselves have arbitrarily added to that essential content given by nature. (Hertz (1899), p.8.)

What began, then, as a challenge to the permissibility of this image is connected in the end to problems with its appropriateness—Hertz regarded the logical tension in the traditional formulation as stemming from inconsistencies in the *inessential* features of the image. Here we have a further indication of the importance Hertz attached to clearly identifying essential features, and pruning down inessential features as far as possible. Crucially, what also comes into view at this point is what Hertz thought his novel reformulation of mechanics could achieve:

> Perhaps our objection is not at all with the contents of the outlined image, but rather only with the form of their representation. We are certainly not too severe if we say that this representation has never attained complete scientific perfection; it yet lacks quite sufficiently sharp distinctions to distinguish what in the outlined image arises from the laws of our thought, what from experience, and what from our own arbitrary choices … In this sense we grant, along with everyone, the permissibility of the contents of mechanics. But it is required by the dignity and importance of our subject that its logical purity is not only acknowledged with good will, but that a perfect representation would prove it. (Hertz (1899), pp.8–9)

Hertz regarded his reformulation of mechanics as achieving two major things. The first was that it was clear which aspects of his image were included for the sake of each of the three criteria. As already noted, Hertz believed the *correctness* of his image came down to the scope and validity of the fundamental law alone.[30] As for *appropriateness* and *permissibility*, the evaluation of these are interconnected. In Hertz's presentation, the careful introduction of the primitive notions (space, time and mass) and the choice of a specific notational framework (the apparatus of differential geometry), along with his stringent axiomatic-deductive procedure, served to highlight how the framework logically cohered (how Hertz's propositions depended on one another), and where certain choices were being made (what alternative equivalent formulations of the fundamental law were possible, for example). The overall result of this leads to the second, and most important, achievement of the book—establishing the logical permissibility of mechanics beyond doubt. Indeed, Hertz strenuously emphasised that clarifying the logical structure of mechanics was his fundamental aim in writing *Principles*:

> I think that as far as logical permissibility is concerned [the image of mechanics I have presented] will be found to satisfy the most rigid requirements, and I trust that others will be of the same opinion. This merit of the representation I consider to be of the greatest importance, indeed of unique importance. (Hertz (1899), p.33)

Thus we see that the sustained polemic challenging the clarity of the logical foundations of the traditional image of mechanics was central in Hertz's motivations. To return to Hertz's preface, we have further clear confirmation of this fact:

> In the details I have not brought forward anything that is new and which could not be found in many books. What I hope is new, and to which alone I attach value, is the arrangement and

[30] Hertz's evaluation of his success in this regard has been disputed; for some discussion see Lützen (2005), p. 132.

presentation of the whole, and thus the logical, or, if one wants, the philosophical aspect of the matter. My work has accomplished its objective or failed insofar as it has gained something in this direction or not. (Hertz (1899) xxiv)

6. "Descending to the world of atoms"

We now need to address the passages in *Principles* where Hertz seemed to indicate that his aim was, after all, to lay the groundwork for an eventual ether theory in precisely the "mechanistic" sense of most of his contemporaries. As noted above (in section 4), at the end of his introduction Hertz considers the plausibility of distant forces compared with rigid connections, seeming to make a direct appeal to developments in electromagnetism—and the concept of an ether—in support of his own formulation of mechanics:

> the balance of evidence will be entirely in favour of the [Hertzian formulation] when a second approximation to the truth can be attained by tracing back the supposed actions-at-a-distance to motions in an all-pervading medium whose smallest parts are subjected to rigid connections; a case which also seems to be nearly realised in the [sphere of electric and magnetic forces]. This is the field in which the decisive battle between these different fundamental assumptions of mechanics must be fought out. (Hertz (1899), p.41)

To make sense of these remarks we need to note that this passage occurs in the concluding paragraph of the introduction (pp.40–41), a paragraph in which Hertz takes an entirely different stance from his discussion up until that point.[31] Earlier, Hertz had been concerned to bring out the difficulties the Newtonian image faced with regard to its permissibility and its appropriateness, and had had no issue at all with its *correctness* (indeed, he remarked 'No one will deny that within the whole range of our experience up to the present the correctness is perfect', Hertz (1899), p. 9). Here, at the conclusion of his introduction, Hertz turns this on its head:

> We shall put the [Newtonian] and [Hertzian] images on an equality with respect to permissibility, by assuming that the first image has been thrown into a form completely satisfactory from the logical point of view … We shall also put both images on an equality with respect to appropriateness, by assuming that the first image has been rendered complete by suitable additions, and that the advantages of both in different directions are of equal value. We shall then have as our sole criterion the correctness of the images. (Hertz (1899), p.40)

Thus the appeal to the concept of the ether that follows is in an extremely hypothetical context. Hertz is assuming that a project *analogous to his own in Principles* has been completed on behalf of the Newtonian image, so that it can be regarded as on a level with the Hertzian image in terms of its permissibility and appropriateness. For such a reformulation of the Newtonian image to be successful, it would have to remove the obscurities concerning 'force' that Hertz took himself to have circumvented in *Principles*. Hence Hertz does not characterize the essential difference between these images in terms of a preference for distant forces over connections or vice versa here. Rather:

> if we try to express as briefly as possible the essential relations of the two representations, we come to this. The [Newtonian] image assumes as the final constant elements in nature the relative accelerations of the masses with reference to each other: from these it incidentally deduces approximate, but only approximate, fixed relations between their positions. The [Hertzian] image assumes as the strictly invariable elements of nature fixed relations between the positions: from these it deduces when the phenomena require it approximately, but only approximately, invariable relative accelerations between the masses. (Hertz (1899), p.41)

In the final analysis, Hertz claims that his own image assumes exact relative displacements, whereas the Newtonain image (if it can be reformulated in a logically perspicuous way) assumes exact relative accelerations.[32] Hertz points out it is likely that only one of these will seem plausible in the light of future accumulated data. Hence, in *this* context, Hertz notes that developments in electromagnetism speak in favour of exact relative displacements over exact relative accelerations, and hence (so the thought goes) future physics may indeed vindicate the Hertzian image. For this situation to arise, the Newtonian image would first have to be reformulated, and results from experimental physics would have to make significant strides forward. But Hertz's project in *Principles* is prior to all this:

> in order to arrive at such a decision it is first necessary to consider thoroughly the existing possibilities in all directions. To develop them in one special direction is the object of this treatise, an object which must necessarily be attained *even if we are still far from a possible decision*, and even if the decision should finally prove unfavourable to the image here developed. (Hertz (1899), p.41, emphasis mine)

As noted, there are two other passages in *Principles* where Hertz refers to 'the world of atoms'. The first is earlier in the introduction, where Hertz responds to the worry that an appeal to connections already assumes the existence of forces. Hertz's interlocutor argues: surely it is precisely the presence of certain forces that maintains such fixed connections. To this Hertz replies, 'Your assertion is correct for the mode of thought of ordinary mechanics, but it is not correct independently of this mode of thought; it does not carry conviction to a mind which considers the facts without prejudice and as if for the first time' (Hertz (1899), p. 34). His point is that there is no need to account for a fixed spatial relation between masses by appeal to forces if one is not already committed to the primacy of the latter. But Hertz's interlocutor pursues the matter, pointing out that all observed rigid connections in nature are only approximate, 'and the appearance of rigidity is only produced by the action of the elastic forces which continually annul the small deviations from the position of rest' (ibid). Hertz replies as follows:

> In seeking the actual rigid connections we shall perhaps have to descend to the world of atoms. But such considerations are out of place here; they do not affect the question whether it is logically permissible to treat of fixed connections as independent of forces and precedent to them. (Hertz (1899), p.34)

In the light of the previous discussion, we can see that Hertz's remarks here do not force the reading that his aim in *Principles* was to lay foundations for an ether mechanism. Note that this is

[31] Some commentators have noted this fact before, including Nordmann (1998), p. 163 and Lützen (2005), p. 118. (As Lützen puts it, 'the last two pages of the introduction read more as a second thought than as a conclusion'.) To my knowledge the only extended discussion of the new stance that Hertz adopts in these concluding passages is in Preston (2008b). However, my assessment of the significance of these passages differs from Preston's.

[32] This point is noted in Nordmann (1998), p. 161 f.

compatible with Hertz's speculation that exact relative displacements may indeed be found at atomic length scales. Nevertheless, Hertz is unambiguous in stating that 'such considerations are out of place here'.[33]

The final passage in which Hertz refers to the "world of atoms" occurs at the end of chapter two of book two:

> in all connections between sensible masses which physics discovers and mechanics uses, a sufficiently close investigation shows that they have only approximate validity, and therefore can only be derived connections. We are compelled to seek the ultimate connections in the world of atoms, and they are unknown to us. (§330)

This is, again, an accommodation of the fact that all observed rigid connections have so far turned out to be approximate. However, this section of the text (§§327–330) in fact highlights the way in which Hertz's project must be regarded as *separate* from an investigation into facts at atomic length scales. Here is how the passage just quoted continues:

> But even if [the ultimate connections in the world of atoms] were known to us we could not apply them to practical purposes, but should have to proceed as we now do. For the complete control over any problem always requires that the number of variables should be extremely small, whereas a return to the connections amongst the atoms would require the introduction of an immense number of variables. (§330)

Hertz points out that even if we *were* confident in our knowledge of phenomena in the atomic domain, it wouldn't change our approach to mechanical problems at larger length scales. For in the treatment of any problem (at any length scale), the free variables have to be kept to a workable number. Indeed, it is a key feature of Hertz's formulation that he can explain clearly how his fundamental law can be applied to systems in *ignorance* of the microscopic details. Recall that a system's connections identify a lower-dimensional hypersurface within its $3n$-dimensional configuration space.[34] In general, one can apply the full apparatus of Hertz's mechanics as soon as one has identified equations of condition of the right form. Hertz makes clear that in applying the fundamental law it doesn't matter at all whether these equations represent underlying connections between the fundamental constituents of the system:

> If we know from experience that a system actually satisfies given equations of condition, then in applying the fundamental law it is quite indifferent whether these connections are original ones, i.e. whether they do not admit of a further physical explanation … or whether they are connections which may be represented as necessary consequences of other connections and of the fundamental law. (§328)

Hertz argues that his own formulation of mechanics simply makes perspicuous the fact that every application of mechanics at ordinary length scales abstracts away from the underlying microscopic details. This point is of fundamental importance in understanding the role of Hertz's hidden masses. As should now be emerging, their role in Hertz's framework is *not* to function as a proposal for the underlying microscopic constituents of systems. The most immediate role of the hypothesis of hidden masses is that it allows Hertz to accommodate the motion of unfree systems within his analytical framework. However, it also plays another crucially important role. Rather than being a proposal concerning the microscopic constituents of systems, the hypothesis of hidden masses *rules out* knowledge of the "fundamental" constituents of a system. This is because the only knowledge of a system that Hertz's mechanics delivers is the existence of a 'dynamical model' of that system:

> If we admit generally and without limitation that hypothetical masses (§301) can exist in nature in addition to those which can be directly determined by the balance, then it is impossible to carry our knowledge of the connections of natural systems further than is involved in specifying models of the actual systems. We can then, in fact, have no knowledge as to whether the systems which we consider in mechanics agree in any other respect with the actual systems of nature which we intend to consider, than in this alone, that the one set of systems are models of the other. (§427)

It is important to appreciate how abstract such dynamical models are. Hertz calls two systems dynamical models of one another if it is possible to write down analytical representations of them which have: (i) the same number of coordinates, (ii) the same equations of condition, and (iii) the same expressions for the magnitude of a displacement (cf. §418). Thus, for instance, any rigid system is a dynamical model of any other. The same applies to any system modeled as a simple harmonic oscillator—a mass on a spring, a pendulum, and a vibrating string are all dynamical models of one another. Indeed, 'An infinite number of systems, quite different physically, can be models of one and the same system. Any given system is a model of an infinite number of totally different systems' (§421). Thus it is built into Hertz's framework that the true "composition" of a material system is radically underdetermined.[35]

Note, here, the close relationship between Hertz's discussion of dynamical models and the image-theory of his introduction. When Hertz introduced the notion of an "image", he posited one fundamental requirement: the consequences of the image in thought must give rise to images of the consequences of the pictured objects. This requirement was an important *limitation* on how our images can represent things in the world: 'we do not know, and we have no way to learn, whether our conception of things conforms with them in any other way, except in this *one* fundamental respect alone' (Hertz (1899), p. 1). With the hypothesis of hidden masses Hertz has shown how this requirement on images in general applies to the images provided by mechanics in particular. Hence it is in the discussion of dynamical models that Hertz makes his only explicit reference back to the general image-theory of his introduction:

> The relation of a dynamical model to the system of which it is regarded as the model, is precisely the same as the relation of the images which our mind forms of things to the things themselves… The agreement between mind and nature may therefore be likened to the agreement between two systems which are models of one another. (§428)

[33] Though I do not explore this idea in detail here, it is clearly relevant that Hertz saw a clear separation between theoretical mechanics and experimental physics—cf. Hertz (1899), p. 27: 'To investigate in detail the connections of definite material systems is not the business of mechanics, but of experimental physics'.

[34] As noted in section 3, this is only strictly true for holonomous connections.

[35] Among other places this point emerges in §536, where Hertz notes that it is 'permissible though arbitrary' to regard any material system whatsoever as composed of some number of coupled subsystems.

7. Conclusion

We began with the curious historical situation that followed the publication of Hertz's book. On the one hand, Hertz's contemporaries regarded *Principles* as a remarkably impressive work; on the other hand, they struggled to identify what it was that Hertz thought he had achieved in writing it. Formulating mechanics by eschewing actions-at-a-distance in favour of hidden masses and connections was all well and good, they thought, but without specifying how mechanisms of hidden masses could plausibly account for observed phenomena in concrete cases, the project was, as Boltzmann put it, doomed to be 'only of purely academic interest' or, at best, 'a programme for the distant future' (Boltzmann (1974), p. 90).

Our task was thus to explain the absence of mechanisms in Hertz's book, and explain why Hertz seemed unperturbed by the difficulties of constructing such a mechanism. The path to the answer involved exploring the significance of Hertz's image-theory of representation, thereby reconstructing his rationale for distinguishing between the essential and inessential elements of a scientific theory. This brought out Hertz's commitment to distilling out the bare *image* of mechanics, and separating this off from any inessential elements that attach to it via the *imaginative* aids we might employ in fleshing it out. Hence we saw that developing the kinds of mechanisms that Hertz's readers looked for would have been anathema to Hertz's intentions: in identifying the essential content of mechanics, he intentionally avoided making any appeal to constructible models or imaginative pictures.[36]

I have claimed that Hertz's own presentation can seem particularly misleading on this issue, especially with regard to his introduction of *Massenteilchen* and hidden masses. A primary concern of this paper has been to show that, rather than being speculative ontological posits, the introduction of such objects served the role of allowing Hertz to formulate suitably *abstract* descriptions of mechanical systems in the form of dynamical models.[37] Hence I have argued that the core value of Hertz's project is not tied to the prospects of finding a suitable ether mechanism. Furthermore, I have argued that Hertz's hypothesis of hidden masses in fact *ruled out* knowledge of the "fundamental" constituents of a mechanical system.

It would be too strong to claim that Hertz did not have *any* expectations concerning how his work would relate to future investigations of the ether. And it is also clear that Hertz did regard the image of mechanics presented in *Principles* as tied to the empirical claim that actions-at-a-distance could ultimately be accounted for in terms of contact actions (cf. §469 in particular). Indeed, Hertz was aware that this was speculative, and acknowledged that future experimental evidence might 'finally prove unfavourable to the image here developed' (Hertz (1899), p. 41). Nevertheless, the core value of *Principles* was, and remains, independent of these issues.

[36] At any rate, this accounts for the lack of examples in *Principles* itself, independently of what Hertz's attitude may have been towards later attempts to develop a Hertzian mechanism.

[37] In this vein it is important to bear in mind that Hertz never intended *Principles* to replace existing approaches to mechanical problems: 'In respect of [practical applications or the needs of mankind] it is scarcely possible that the usual representation of mechanics, which has been devised expressly for them, can ever be replaced by a more appropriate system' (Hertz (1899), p. 40). See also Lützen (2005), p. 263: 'Since [Hertz] could show that the usual principles of mechanics also hold in his image of mechanics any analysis of a mechanical problem within the usual mechanics is, in a sense, also valid in his mechanics'.

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