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# LUDWIG BOLTZMANN'S *BILDTHEORIE* AND SCIENTIFIC UNDERSTANDING

ABSTRACT. Boltzmann's *Bildtheorie*, which asserts that scientific theories are 'mental pictures' having at best a partial similarity to reality, was a core element of his philosophy of science. The aim of this article is to draw attention to a neglected aspect of it, namely its significance for the issue of scientific explanation and understanding, regarded by Boltzmann as central goals of science. I argue that, in addition to being an epistemological view of the interpretation of scientific theories Boltzmann's *Bildtheorie* has implications for the nature of scientific understanding. This aspect has as yet been ignored because discussion of the *Bildtheorie* has been restricted to the realism-instrumentalism debate. To elucidate my analysis of Boltzmann's *Bildtheorie* concrete examples are presented, and the pragmatist and Darwinist roots of Boltzmann's view are discussed.

Moreover, I propose to use Boltzmann's ideas as a starting-point for developing a novel analysis of the notion of scientific understanding, of which a brief impression is given. It shows that the study of Boltzmann's philosophy is not only of historical interest but can be relevant also to modern philosophy of science and to the methodology of theoretical physics.

## 1. INTRODUCTION

Boltzmann's *Bildtheorie* of scientific knowledge has been discussed extensively in the literature, notably by Hiebert (1981), Miller (1986), Wilson (1989, 1993), D'Agostino (1990), and Blackmore (1995b). In these works the *Bildtheorie* has been examined, almost as a matter of course, in the context of the epistemological question of whether (and if so, how) scientific theories are representations of reality. Of course, this was also the viewpoint from which Boltzmann himself usually presented his *Bild*-conception of scientific theory. However, an important, arguably even more fundamental, issue has received scant attention. According to Boltzmann, a central aim of science was to give explanations of natural phenomena – in contrast to merely describing and predicting observable phenomena (see, e.g., Boltzmann 1974, 16, where he argues against Kirchhoff's views). In his 1902 inaugural address 'On the principles of mechanics'. Boltzmann (1974, 149) expressed his view as follows: "It is the ubiquitous task of science to explain the more complex in terms of the simpler; or, if pre-

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ferred, to represent [*anschaulich darstellen*] the complex by means of clear pictures [*Bilder*] borrowed from the sphere of the simpler phenomena".[1] This statement shows that in Boltzmann's philosophy of science *Bilder* were essential to the explanatory aim of natural science.

In this paper I will investigate in detail how precisely *Bilder* fulfill their explanatory task; or, in other words, how they contribute to the scientific understanding of natural phenomena. First, in Section 2, I will review Boltzmann's *Bildtheorie* and the way in which it is commonly analysed in the literature, namely as a particular epistemological view of the relation between scientific theories and reality. Section 3 presents an analysis of the hitherto neglected role of *Bilder*, namely as tools for scientific understanding. After investigating Boltzmann's ideas concerning explanation and understanding, I outline how these can be employed to construct a new analysis of the notion of understanding. I illustrate my approach by showing how an atomistic picture of matter generates scientific understanding. Finally, in Section 4, I argue that Boltzmann's pragmatism, a trait of his philosophy which has not yet been deeply investigated, is fundamental to his view of explanation and understanding.

## 2. BOLTZMANN'S *BILDTHEORIE* AND ITS RECEIVED VIEW

Most of Boltzmann's remarks regarding the aims and the nature of scientific theories can be found in his *Populäre Schriften*, containing philosophical essays he wrote in his later years. In this section I briefly review some important passages in which Boltzmann presents and defends his conception of scientific theories, known as the *Bildtheorie*. The earliest formulation can be found in his 1890 address 'On the Significance of theories', where Boltzmann (1974, 33) stated that the "task of theory consists in constructing a picture of the external world that exists purely internally and must be our guiding star in all thought and experiment". His 1899 essay 'On the Development of the Methods of Theoretical Physics in Recent Times' contains a more elaborate exposition of the *Bildtheorie*, which Boltzmann (1974, 90–91;) traces back to Maxwell and Hertz: "Hertz makes physicists properly aware of something philosophers had no doubt long since stated, namely that no theory can be objective, actually coinciding with nature, but rather that each theory is only a mental picture of phenomena, related to them as a sign is to designatum".[2] He adds that this implies that "it cannot be our task to find an absolutely correct theory but rather a picture that is as simple as possible and represents the phenomena as accurately as possible".

These statements of the *Bildtheorie* are well-known, and discussed by many philosophers and historians of science, among whom Hiebert (1981), Miller (1986), D'Agostino (1990), Wilson (1989, 1993), Blackmore (1995b, 69–74) and De Regt (1996). All these writers focus on the realism-instrumentalism debate and attempt to assign a position to Boltzmann in the associated spectrum of standpoints. Clearly, his philosophical views were – at least in his later years – rather sophisticated, as he did not unconditionally assert the reality of atoms but neither completely denied their existence. This led D'Agostino (1990, 383) to conclude that with the *Bildtheorie* "Boltzmann succeeds in obtaining a remarkable balance between the opposite pretensions of crude phenomenology and crude (ontological) atomism". According to Hiebert (1981, 180), Boltzmann combined philosophical idealism with pragmatic realism. Blackmore (1995a, 27–36) presents an overview of the various views on Boltzmann's philosophy, and his evaluation leads him to endorse the view that Boltzmann, like so many physicists of his day, took a "split position" (ibid., 36): on the one hand he employed phenomenalist language, while on the other hand he accepted the reality of atoms and molecules in actual scientific practice. In his second Boltzmann-volume, Blackmore (1995b, 69–74) discusses the *Bildtheorie* in more detail, emphasizing that Boltzmann denied atoms ontological status and that he considered the atomic pictures useful idealizations that require a non-correspondence theory of truth. In a previous article (De Regt 1996, 41–43), I have attempted to analyse Boltzmann's philosophy by sharply distinguishing between an epistemological and a methodological part, which combine to a form of 'constructive realism'.

An interesting early reference to the *Bild*-conception of physical theory, that has not yet received attention in the secondary literature, can be found in the first volume (1891) of Boltzmann's *Vorlesungen über Maxwells Theorie der Elektricität und des Lichtes.* In the second lecture, Boltzmann discusses a mechanical analogy of the second law of thermodynamics to illustrate the physical significance of the Lagrangian equations of motion. Having explicated the analogy, Boltzmann (1982, vol. 1, 13–14) concludes the lecture with a section 'Die Theorien sind blosse Bilder der Naturprocesse' [Theories are mere pictures of natural processes]. He states that the sketched analogy also serves to elucidate the nature of the theories to be discussed in his *Vorlesungen*, and indeed of many physical theories. They do not pretend to give true representations of states of affairs and processes in nature, but only to describe mechanisms that have a strong analogy with natural phenomena. Boltzmann reminds us of Maxwell's 'dynamical illustrations' and emphasizes the advantages and heuristic value of their

clear and well-defined character. In the second volume (1893) of the *Vorlesungen*, Boltzmann (1982, vol. 2, 22) introduces a distinction between two different interpretations of mechanical pictures. Firstly, a physical theory in itself can be regarded as a picture, as an *Analogon.* Secondly, one may employ specific mechanical analogies in order to obtain visualizations (*Versinnlichung*) of the consequences of a theory.[3] This amounts to distinguishing between epistemological and methodological roles of *Bilder*, which relate to their representative and understanding-generating functions, respectively (see below, Section 3.1).

An important subject of discussion in the secondary literature on the *Bildtheorie* is the relation between Boltzmann's and Hertz's views. Although Boltzmann frequently referred to Hertz when stating the *Bildtheorie*, Wilson (1989) and D'Agostino (1990) have argued that there are important differences between Boltzmann and Hertz. First of all, Boltzmann rejected Hertz's demand that the mental pictures must be laws of thought. While Hertz's view on laws of thought was thoroughly Kantian, Boltzmann argued – against Kant – that laws of thought may exist and even be innate but always can be modified by education and experience. He concluded: "I therefore wish to modify Hertz's demand and say that insofar as we possess laws of thought that we have recognized as indubitably correct through constant confirmation by experience, we can start by testing the correctness of our pictures against these laws; but the sole and final decision as to whether the pictures are appropriate lies in the circumstance that they represent experience simply and appropriately throughout so that this in turn provides precisely the test for the correctness of these laws" (Boltzmann 1974, 105; cf. 137, 165–6 and 195; see also Section 3.2 of the present paper).

According to Wilson (1989, 254-55), another essential difference between Boltzmann and Hertz is that for Boltzmann the main characteristic of *Bilder* is their visualizability: "For Boltzmann, physically visualizable (*anschaulich*) representations come first, followed by mathematical formalism". By contrast, Hertz valued first and foremost the logical and mathematical form of theoretical pictures. Wilson concludes that "Boltzmann was in no way a Hertzian". Given the many times that Boltzmann approvingly mentioned Hertz, this conclusion appears exaggerated. Further evidence against Wilson's claim can be found in Boltzmann's *Vorlesungen über Maxwells Theorie*, where, as we saw above, a distinction was introduced between considering the whole theory as an analogical picture and employing visualizations to illustrate particular consequences of the theory. Only in the latter case does the picture have to truly visualizable. That Boltzmann also intended the *Bildtheorie* to function in the former

sense is confirmed by the fact that at various places he argued that not only atomism but also Kirchhoff's mathematico-physical phenomenology is a mental picture (see Boltzmann 1974, 42 and 96).

A final point of interest is the fact that Boltzmann had conceived of the *Bildtheorie* already before he read Hertz's work. Wilson (1989 and 1993) argues that the real sources of Boltzmann's views were James Clerk Maxwell and Robert Zimmermann, his philosophy teacher at Vienna University. Indeed, Boltzmann advanced the *Bildtheorie* as early as 1890, four years before Hertz's *Prinzipien der Mechanik* appeared, and also in 1891, in the *Vorlesungen über Maxwells Theorie*, citing Maxwell as his source. In later years, however, Boltzmann most often referred to Hertz when he introduced the *Bild*-conception (e.g., Boltzmann 1974, 214 and 225). Incidentally, Wilson's finding that the roots of Boltzmann's epistemological position lie in his early education lends additional support to my thesis concerning the heuristic role of Boltzmann's philosophy (De Regt 1996, 56). From a study of Boltzmann's work on the kinetic theory I concluded that the core elements of his mature philosophy were already present in his early years, and that his epistemological and methodological ideas guided his attempts to solve the specific heat anomaly. Following the standard view that Boltzmann took the *Bildtheorie* from Hertz, I faced the problem that direct evidence for his early philosophical position is lacking (De Regt 1996, 40). However, Wilson has provided indirect evidence that reinforces the conclusion drawn in my study.

## 3. *BILDER* AS TOOLS FOR SCIENTIFIC UNDERSTANDING

### 3.1. *Representative versus Explanatory Function of Bilder*

As we saw in the previous section, Boltzmann's *Bildtheorie* is usually regarded as an answer to the epistemological question of whether, and if so in which respects, scientific theories correspond to reality. It is, in other words, taken to be a specific view concerning the representative character of scientific theories. In this section I want to highlight another aspect of the *Bildtheorie*: the role of *Bilder* in the activity of scientific explanation; their function as tools for scientific understanding.

Of course I do not claim that discussions of the representational function of *Bilder* are beside the point. It cannot be doubted that the *Bildtheorie* is first of all a thesis about representation of reality. Indeed, Boltzmann himself often presented it as such (see Blackmore 1995b, 70). This fits in with the historical context: the representational question was central to late-nineteenth-century debates between defenders of the mechanical-

atomistic worldview – among whom was Boltzmann – and proponents of alternative conceptions of nature (phenomenalism, energetics, electromagnetic worldview). Boltzmann certainly intended his *Bildtheorie* as a sophisticated answer to the phenomenalists, who charged the atomists with naive realism and who defended the view that theories should not at all pretend to describe unobservable reality.

However, the controversy between Boltzmann and the phenomenalists (e.g., Mach and Kirchhoff) did not only concern the epistemological interpretation of theories (realism vs. instrumentalism), but also the issue of whether the aim of science is explanatory or descriptive. While Mach and Kirchhoff considered the aim of science to be mere (economical) description of observable phenomena, Boltzmann held that science must strive after explanations of the phenomena.[4]

In the case of atomism, this means that the question is not only whether the atoms postulated by the kinetic theory really exist, or have to be regarded merely as abbreviations to describe and predict observable phenomena (e.g., the macroscopic behaviour of gases). Boltzmann's answer was, as we have seen, that the kinetic theory does not pretend to give a literally true representation of reality but provides a *Bild* that possesses a certain similarity with the unobservable reality. However, an additional question is whether atomic theory gives a better understanding of the behaviour of gases than phenomenological thermodynamics. If the answer to the latter question is 'yes', as Boltzmann claimed, then wherein lies this difference in explanatory power? Boltzmann's *Bildtheorie* blocks the simple and straightforward answer of the die-hard realist, to wit that the kinetic theory provides understanding because it describes the 'deeper truth' about the constitution of gases. But how can a scientific theory provide understanding in a different way?

In Section 1, I quoted Boltzmann on the aim of science: "It is the ubiquitous task of science to explain the more complex in terms of the simpler; or, if preferred, to represent the complex by means of clear picture-borrowed from the sphere of the simpler phenomena". Thus, Boltzmann evidently considered explanation a central aim of science; and he suggested that explanation of a phenomenon is possible by developing a theory of it which employs *anschauliche Bilder* ("clear pictures borrowed from the sphere of the simpler phenomena"). Following this lead, I will show how the sophisticated epistemology of the *Bildtheorie* leads to a similarly sophisticated theory of explanation and understanding.[5]

### 3.2. *Boltzmann's View of Scientific Understanding*

Having asserted that the aim of science is explanation, Boltzmann observed (in the passage following the statement quoted above) that this induced physicists to develop mechanical explanations of various physical phenomena. Why mechanical? Because mechanics provides *anschauliche Bilder* par excellence. The simplest conceivable phenomena consist of particles in motion, and therefore one tried to explain all phenomena by means of the science of motion: mechanics. At various places, Boltzmann suggested that he held mechanical explanations to be the most intelligible and therefore the most satisfying ones. For example, in his 1900 inaugural address 'On the Principles of Mechanics' he stated:

> If [ ... ] all apparently qualitative changes were representable by the picture of motions or changes of arrangements of smallest parts, this would lead to an especially simple explanation of nature. In that case nature would appear to us at its most comprehensible [*begreiflichsten*], but we cannot compel her to this, we must leave open a possibility that this will not do and that we need in addition other pictures of other changes; understandably, it is precisely the more recent developments of physics that have made it prudent to allow for this possibility. (Boltzmann 1974, 143)

This statement seems to carry the idea that there exist a priori criteria for assessing when a scientific theory provides understanding; criteria which imply that theories giving mechanical pictures provide maximal understanding because of their simplicity.[6] Such arguments may give the impression that Boltzmann's preference for mechanical explanations was based on a Kantian position, according to which our intellectual faculties or immutable laws of thought determine the form of our scientific theories. However, this impression would be mistaken: as mentioned in Section 2, D'Agostino (1990) and others have demonstrated that Boltzmann was certainly not a Kantian. Indeed, in a passage immediately preceding the above quotation, Boltzmann (1974, 142–43) stated that he did not think highly of attempts to give a priori proofs of mechanicism. His opposition to Kantianism is especially apparent in his rejection of Hertz's view of *Bilder* as corresponding to 'laws of thought'.

What reasons could Boltzmann have had to prefer mechanical pictures as being most intelligible, if there were no a priori arguments for it? In order to answer this question we must turn to Boltzmann's views with respect to laws of thought (*Denkgesetze*), which he proposed as an alternative to the ideas of Hertz and Kant. As D'Agostino (1990, 388–90) has observed, Boltzmann adhered to an evolutionary, Darwinist conception of laws of thought. On this view, there do indeed exist laws of thought which are in some cases innate and in this sense a priori. However, these laws have achieved this status through their evolutionary success. They have

become innate and apparently immutable because of their lasting utility in the struggle for survival. But this does not imply that such laws are absolutely infallible or a priori in the traditional sense: there may always turn up new, unexpected situations in which they fail.[7] As will be argued below, Boltzmann held that explanation through mechanical pictures corresponds with our laws of thought in this evolutionary sense and is therefore preferable but not infallible.

An extensive discussion of the nature and status of *Denkgesetze* can be found in Boltzmann's Lectures on Natural Philosophy (1903– 1906), published in 1990 by Ilse Fasol-Boltzmann as *Principien der Naturfilosofi.* [8] In the first course Boltzmann analysed elementary concepts such as numbers, space, and time. He defended the thesis that even these simple and fundamental notions are not a priori given but related to experience. In his notes for the third lecture, Boltzmann remarked that we should not designate the notion of number as being a priori in the sense of 'true independently of the world' but ask ourselves if it is useful (*zweckmässig*) to regard it as such. He approvingly cited the thesis of Kant and Schopenhauer that we need a priori notions in order to be able to think (in other words, that laws of thought must exist), but added that this idea should be connected with Darwin and that a priori ideas are not necessarily correct (see Fasol-Boltzmann, 1990, 84). In the *Mitschrift* of this lecture we can read how Boltzmann presented these ideas to his students (Fasol-Boltzmann 1990, 160). He acknowledged the existence of a priori ideas (*aprioristische Vorstellungen*) and emphasized that they were necessary for obtaining knowledge ('constructing the world out of experiences'). Their presence can be explained by Darwin's theory: they are inherited from our ancestors and have developed in interaction with the environment. This implies, however, that such ideas are not necessary truths.

Boltzmann's use of the term a priori is thus somewhat idiosyncratic. Traditionally, a priori propositions are propositions known to be true (or false) without reference to experience, and a priori concepts are concepts that are not derived from experience. For Boltzmann the term a priori is equivalent to 'innate'. This is related to the traditional meaning but not completely identical to it, because of Boltzmann's Darwinist view: an idea can be a priori in the sense that it is independent of the experience of an individual (being innate it precedes the experiences of the individual); however, it is not fully independent of experience because it is dependent on (formed by) the experiences of the ancestors of the individual in question. Just as evolutionary processes can change any innate property of a species, it can also change the a priori ideas of the human species.

In the fourth lecture Boltzmann reiterated his evolutionary conception of *Denkgesetze*, and proceeded to argue explicitly against Hertz's view of the relation between scientific theories and laws of thought. Hertz held that "firstly, and most importantly, a theory should conform to the laws of thought, and secondly it should accord with experience" (Fasol-Boltzmann 1990, 168). Boltzmann agreed with this prescription if taken as a practical guideline. Indeed, Hertz's advice appears to be similar to Boltzmann's own hypothetico-deductive methodology, which he had baptized 'deductive representation': *deduktive Darstellung* (Boltzmann 1979, 165, and 1974, 107–8). According to this methodology one should beware of bringing in experience too early in the process of theory construction, because this would lead to lack of clarity and generality (see De Regt 1996, 43–5). However Boltzmann fiercely objected to giving Hertz's advice the status of a universal epistemological principle. He held, by contrast, that experience has in the end more epistemic weight than laws of thought: "It is possible that a theory disagrees with he laws of thought in some respect, while it does accord with nature; in that case the law of thought should be modified" (Fasol-Boltzmann 1990, 168).

This is of course more easily said than done. Because laws of thought are innate and have been useful in previous situations, we often cling strongly to them even when they are no longer applicable. In that case the laws of thought 'overshoot the mark' (*schiessen über das Ziel hinaus*), to use a favourite expression of Boltzmann: "I would deny that our laws of thought are absolutely perfect. On the contrary, they have become such firmly established habits that they overshoot the mark and will not let go even when they are out of place" (Boltzmann 1974, 195). However, in the long run it will often be possible to abandon them. For example, nowadays non-Euclidean geometry is understood by many educated people, while for centuries Euclidean geometry was regarded as the only intelligible possibility, entailed by laws of thought (Fasol-Boltzmann 1990, 288, cf. 283).

Returning to the topic of scientific explanation, we now see why Boltzmann favoured mechanical explanation. Although he did not believe in the traditional notion of immutable laws of thought (allegedly necessitating mechanical explanation), he claimed that there do exist laws of thought formed in the evolutionary sense sketched above. These laws have made explanation through mechanical pictures a preferable mode of explanation, albeit not an infallible one. Accordingly, the intelligibility of mechanical pictures is only a contingent fact. It is the practical success of mechanicism – possibly linked with our familiarity with mechanical

systems from daily experience – that has made it into a criterion for intelligibility in science.

In the quotation at the beginning of this subsection, Boltzmann (1974, 143) said that we cannot compel nature to being explicable by means of mechanical pictures. He added that "more recent developments in physics" illustrate this situation. Here he hinted at the fact that at the end of the nineteenth century the project of mechanical explanation met with difficulties, which induced physicists to search for alternatives. One of these was the electromagnetic worldview, based on the assumption that mechanical phenomena should be explained on the basis of electromagnetic conceptions instead of the other way around (e.g., explaining inertial mass from the motion of charges). On this view the fundamental electromagnetic entity – the ether – served as the most fundamental mechanism that had to explain all physical phenomena. Boltzmann (1974, 150) observed: "It was no longer a question of explaining everything mechanically, but of finding a mechanism to explain all mechanisms". This reversal of the direction of explanation led him to reflect on the question of what it means to have understanding of a mechanism:

> What, then, is meant by having perfectly correct understanding of a mechanism? Everybody knows that the practical criterion for this consists in being able to handle it correctly. However, I go further and assert that this is the only tenable definition of understanding a mechanism. (Boltzmann 1974, 150)

This is a thoroughly pragmatic definition of understanding. Boltzmann appeared to have accepted the full consequences of his evolutionary view of laws of thought. Although laws of thought, and thus criteria for understanding, exist, they are products of evolutionary selection processes and therefore grounded in practical success. From this it is only a small step to defining understanding in the above pragmatic manner. In Section 4 I will elaborate on Boltzmann's pragmatism in relation to the issue of scientific understanding.[9]

Notwithstanding this relativization of the status of mechanical explanations, Boltzmann still hoped and believed that mechanical pictures would finally be victorious when it came to assessing them on their fruitfulness. Thus, he concluded his 1900 address with the following words: "Summing up in conclusion, our result is that one side of all processes of inanimate and animate nature is representable through purely mechanical pictures, or, as the phrase goes, made intelligible with a measure of exactness not hitherto achieved in any other way, while at the same time none of the higher endeavours and ideals are in the least impaired" (Boltzmann 1974, 146; translation revised).[10]

### 3.3. *Boltzmann's View: A Prelude to a New Theory of Scientific Understanding*

In the previous subsection I have given what I believe to be a plausible reconstruction of Boltzmann's approach to the issue of scientific explanation and understanding. Of course, Boltzmann did not have a full- fledged philosophical theory of understanding and neither did he attempt to develop one. His philosophical work, as most philosophical writings by natural scientists, was not very systematic. As Blackmore (1995b, 65) rightly remarks, one might even doubt whether Boltzmann had a coherent worldview at all. Nevertheless, I want to suggest that Boltzmann's ideas can be employed to construct a full-fledged theory of scientific understanding. In the present subsection I will sketch such a theory (for a more detailed, though still tentative account, see Dieks and De Regt 1998).

The starting-point of my analysis is the idea that scientific understanding of a phenomenon is achieved if one possesses a theory of it that is both empirically adequate and intelligible. Of course, this immediately leads to the question: When is a theory intelligibility? Extant philosophical views postulate universally valid criteria for intelligiblity (e.g., causality or visualizability). Think, for example, of a Kantian view of laws of thought restricting the class of allowed theories to intelligible ones. The main problem of this approach is that scientific developments are bound to violate any postulated criterion. For example, in the late- nineteenth century the mechanical view of nature confronted serious problems that have in the present century led to quantum mechanics and relativity, theories that fly in the face of Kantian criteria for intelligibility.

In order to evade these problems I defend a more contextual approach to scientific understanding. The core idea of this approach is the following characterization of the notion of an 'intelligible theory': a scientific theory is intelligible if one is able to recognize at least qualitatively its consequences without performing exact calculations.[11] It will be clear that on this interpretation in principle every theory can be intelligible. There are no necessary (a priori) conditions for intelligibility. The definition of intelligibility does not distinguish between theories that are either a priori intelligible or unintelligible. For example, it does not follow from the definition that only theories based on mechanical pictures are intelligible and provide understanding. However, when applying the definition, it might turn out that in a particular community at a specific time in history it is de facto only mechanical theories that fulfill the above characterization. This indicates that intelligibility is a contextual matter: since a scientific theory is intelligible for a particular agent, namely the user of the theory, its intelligibility automatically comes to depend on such contextual factors

as the capacities, background knowledge and background beliefs of this agent.

Rendering a theory intelligible in the above sense requires conceptual 'tools', needed to draw conclusions without calculations. What are these tools? It is obvious that general concepts such as causality and continuity may play a vital role in 'seeing intuitively' the consequences of a scientific theory. More specific tools might be visualizations; think, for example, of field lines in electrostatics, which can be used to obtain a qualitative idea of the direction and strength of the electric field in simple situations (see Feynman et al. 1964, Vol. II, section 4–8). However, many theoretical physicists have developed an intuition for the behaviour of the solutions of the equations they use, which enables them to have a feeling for the quantitative behaviour of the described systems without invoking picturable physical mechanisms. In the present analysis of intelligibility, this is no problem because such different tools as visualizable physical mechanisms on the one hand, and rather abstract reasoning on the other, fit in with one and the same central aim of science, namely achieving scientific understanding. It seems that the many criteria for intelligibility proposed by scientists and philosophers of science can be accommodated within this analysis as 'tools for understanding'. It thus appears that the contextuality of understanding is essentially related to the question of which tools are suitable and available. There is no universal tool for understanding, but only particular tools for particular people in particular situations. Which tools they have at hand and can apply is dependent on the (historical) context in which they find themselves.

It is my contention that this conception of scientific understanding allows us to incorporate Boltzmann's views regarding the status and function *Bilder* into a coherent philosophical theory. Moreover, I submit that this theory gives a plausible account of the way Boltzmann employed *Bilder* in scientific practice (see below). Boltzmann's emphasis on the evolutionary character of laws of thought (such as mechanical pictures, but also abstract concepts like numbers) entails that these are not universal and infallible but contextual and justified by practical success. Indeed, laws of thought are tools which might be replaced by other tools if the new ones prove to be more useful (Gerhard Fasol in Fasol- Boltzmann 1990, 67). The proposed analysis of scientific understanding gives a more specific account of the function of these conceptual tools, namely to make it possible to intuitively see consequences of a theory. This is in line with Boltzmann's declaration that the task of science is to explain complex phenomena with the help of "clear pictures borrowed from the sphere of the simpler phenomena" (Boltzmann 1974, 149).

An interesting feature of the proposed theory is that it uncouples understanding and realism: scientific understanding is possible without being committed to a realist interpretation of the explanatory theory. This runs counter to the realist idea that explanatory power and truth go together. Again, this feature appears to be in the spirit of Boltzmann, whose *Bildtheorie* denied a straightforward realist view of mechanical models and yet regarded them as essential for the explanation of observable phenomena (and thus more than merely a device for the economical description of such phenomena).[12] In Boltzmann's own words: "we invent the atoms for the explanation of the phenomena" (Fasol-Boltzmann 1990, 275). In the lemma 'Model' for the 1902 edition of the *Encyclopaedia Britannica* Boltzmann elucidated the explanatory function of mechanical models in relation to their *Bild*-character in more detail:

> But while it was formerly believed that it was allowable to assume with a great show of probability the actual existence of such mechanisms in nature, yet nowadays philosophers postulate no more than a partial resemblance between the phenomena visible in such mechanisms and those which appear in nature. Here again it is perfectly clear that these models of wood, metal and cardboard are really a continuation and integration of our processes of thought; for, according to the view in question, physical theory is merely a mental construction of mechanical models, the working of which we make plain to ourselves by analogy of mechanisms we hold in our hands, and which have so much in common with natural phenomena as to help our comprehension of the latter. (Boltzmann 1974, 218)

### 3.4. *Example: How Atomistic Bilder Can Provide Understanding*

A simple and general example of a *Bild*, and for Boltzmann arguably the most important one, is atomism: the idea that matter is composed of elementary individuals, to wit indivisible particles. In this subsection I will explain how atomistic pictures can produce scientific understanding in the sense described above. This serves to show that the proposed theory of understanding is in line with scientific practice.

Boltzmann begins his *Lectures on Gas Theory* with an introductory section entitled 'Mechanical Analogy for the Behavior of a Gas'. He sets out by introducing Clausius' distinction between the general theory of heat (phenomenological thermodynamics) and the special theory of heat (the kinetic theory of gases based on the atomistic hypothesis that heat is molecular motion). The latter theory, says Boltzmann (1964, 26), should be considered as a mechanical analogy, a term that indicates "how far removed we are from that viewpoint which would see in visible matter the true properties of the smallest particles of the body". The remaining part of the section is devoted to a purely qualitative analysis that leads to the conclusion that a gas can be pictured as a collection of freely moving molecules in a container.

In a quite straightforward way, this molecular-kinetic picture can give us a qualitative feeling for the behaviour of macroscopic properties of gases. First of all, it follows that heat can be identified with molecular motion and that an increase of temperature corresponds with an increase of the (average) kinetic energy of the gaseous molecules.[13] Moreover, the picture immediately gives us a qualitative explanation of the fact that a gas exerts pressure on the walls of its container. If a moving gas molecule collides with a wall of the container, it gives it a little push. The total effect of the pushing of the molecules produces the pressure. In more formal, mechanical terms: a molecule exerts a force on the wall and the total force of all molecules on a unit area equals the pressure.

With these interpretations we can obtain, furthermore, a qualitative understanding of the relations between temperature, pressure and volume of a gas. If one adds heat to a gas in a container of constant volume, the average kinetic energy of the moving molecules – and thereby the temperature – will increase. Moreover, this means that on the average the velocities of the molecules increase, and that therefore they will hit the walls of the container more often and with more force. In other words, the pressure of the gas will increase. Thus, we have a qualitative explanation of the fact that in a container of constant volume the pressure of a gas is proportional to its temperature. In a similar manner, we can argue that a decrease of the volume of a gas leads to an increase of its temperature (see, e.g., Feynman et al., 1963, Vol. 1, 1–4). Together these conclusions lead to a qualitative expression of Boyle's ideal gas law. On the view defended above, scientific understanding of a phenomenon is achieved when we have a theory of it that is both empirically adequate and intelligible; and a theory is intelligible if one is able to recognize qualitatively its consequences without performing exact calculations. The kinetic theory of gases, based on the atomic picture of matter, allows us to make such qualitative predictions, e.g., of thermodynamic relations between macroscopic quantities. It is for this reason that one can say that the kinetic theory yields scientific understanding. Even without endowing atoms with specific properties (which has to be done to construct an interesting theory with which precise calculations can be done), the atomic hypothesis provides a *Bild* that can give us understanding of the behaviour of gases.

The reduction of thermodynamics to the kinetic theory is generally regarded as a paradigm case of an explanation relation that increases our understanding. Many philosophers invoke it to promote particular, allegedly universally valid, criteria for intelligibility, such as causality or visualizability. James Cushing (1991, 342–5), for example, argues that there is a difference between 'formal explanation' and 'understanding',

where the latter is achieved only if one has a causal picture of the underlying mechanisms. On his view, deriving Boyle's law from statistical mechanics gives a formal explanation, but only the kinetic theory can provide understanding of it. The reason, he says, is that in the statistical derivation there is "no understanding of the physical mechanism", while in the kinetic derivation there is a "causal picture story" (Cushing, 1991, 344). Cushing does not make it clear, however, why one needs causal pictures to achieve understanding. He merely refers to an "intuition, based on experience and on (some) history of physics, that understanding of physical processes must involve picturable mechanisms and processes that can be pictured" (ibid., 341). Although I agree with Cushing that the kinetic theory provides understanding of Boyle's law, and even that this is connected with its 'causal picture story', I hold that this should be seen in the light of a theory of understanding that differs fundamentally from Cushing's view. On my account, sketched above, visualization and causality should not be regarded as necessary conditions for understanding. Rather, they are tools that are useful in particular contexts. Consequently, my theory of understanding remains applicable when causal picture stories are highly problematic, such as in quantum mechanics.

## 4. BOLTZMANN'S PRAGMATISM, DARWINISM AND UNDERSTANDING

> The task of our thinking is so to use and combine [mental pictures] that by their means we always most readily hit upon the correct actions and guide others likewise. In this, metaphysics follows the most down-to- earth and practical point of view, so that extremes meet. (Boltzmann 1974, 104)

In Section 3 we have seen that Boltzmann proposed a pragmatic definition of 'understanding a mechanism': he argued that there is only one tenable definition of understanding a mechanism, namely "the practical criterion" of "being able to handle it correctly" (Boltzmann 1974, 150). More generally, Boltzmann appears to equate 'to understand' with 'to be able to act correctly in all circumstances'. I have noted already that this pragmatic stance seems to be related to Boltzmann's Darwinism, particularly his evolutionary view of laws of thought. If laws of thought and the criteria for understanding are products of evolutionary selection processes, they are grounded in (justified by) their practical success. While the Darwinistic traits in Boltzmann's philosophy are generally acknowledged, his penchant for pragmatism has gone largely unnoticed. Only John Blackmore (1995b, 78–82) has paid serious attention to the elements of pragmatism in Boltzmann's philosophy of science. Erwin Hiebert (1981, 181) had earlier called Boltzmann a 'pragmatic realist' but did

not relate this to philosophical pragmatism. In the present section, I will investigate Boltzmann's inclination to pragmatism, elaborating on Blackmore's analysis, and argue that it lies at the root of his views on scientific understanding.

Philosophical pragmatism, founded by Charles Sanders Peirce and William James, is first of all a theory of meaning and truth. It was Peirce who first introduced the term 'pragmatism'. In his version of it the central idea is that the meaning of a concept is given by its practical uses and effects. In Peirce's own words: "Consider what effects, which might conceivably have practical bearings, we conceive the object of our perception to have. Then our conception of these effects is the whole of our conception of the object" (Peirce 1878, 293). William James further developed pragmatism and turned it into a theory of truth, which asserts that a doctrine (theory, statement) is true if it has desirable consequences: "An idea is 'true' so long as to believe it is profitable to our lives" (James 1907, 75). I will argue that there are similarities between such pragmatist ideas and the views of Boltzmann. A very clear example is the following remark, which seems an explicit endorsement of Peircian pragmatism: "How we are to form our concepts cannot be defined and is indeed quite indifferent, so long as they always lead to the correct mode of action" (Boltzmann 1974, 150). Whether there has been a direct influence of Peirce and/or James on Boltzmann, for example whether they have met during Boltzmann's travels in the U.S.A., is however dubious. Boltzmann never referred to them in his writings or lectures. According to Blackmore (1995b, 80) there is a possible but as yet unproven influence.

Boltzmann's first step in the direction of pragmatism was taken in the 1890s, when he had to defend atomic theory against the attacks of phenomenalists and energeticists. In these debates Boltzmann argued that atomism was preferable for its practical virtues: theories that employ mechanical pictures are heuristically more fruitful. His strategy was to avoid having to claim that atoms do exist, and to lay emphasis on their indispensability in scientific practice. Several authors have noticed Boltzmann's pragmatically oriented argumentation and have related it in one way or another to the attenuated realism of Boltzmann's *Bildtheorie* (see e.g., D'Agostino 1990, 383–4, and Hiebert 1981, 180–1). On the other hand, the thesis that atoms are merely "a useful picture" might have been just an "extremely cunning" argumentative strategy against the anti-atomists, as Gerhard Fasol claims (Fasol-Boltzmann 1990, 66), and have no implications for Boltzmann's own position regarding the existence of atoms (which according to Fasol was still uncompromisingly realist). We will see, however, that the argu-

mentation can more plausibly be considered as a first step towards the more radical philosophical pragmatism held by Boltzmann after 1900.

Already in 1897, in his essay 'On the Question of the Objective Existence of Processes in Inanimate Nature', Boltzmann had hinted at applying a pragmatic conception of truth to scientific theories:

> Actions that are followed by things we desire and ideas under whose guidance we act in this manner we denote as correct [*richtig*]. We must aim at having ideas that are correct and economical as well, that is we are to be able always to reach the correct mode of action with the least expenditure of time and effort. The demand on any theory is that it be correct and economical. (Boltzmann 1974, 58)

Nevertheless, it was only after the turn of the century that Boltzmann came out in defence of pragmatism more strongly, as first noticed by Blackmore (1995b, 78–82).[14] Blackmore relates this to Boltzmann's view of *Bilder* as idealizations: theoretical physics cannot do without idealizations and therefore it needed a non-correspondence theory of truth. Thus, in 1904 Boltzmann stated: "We call these mental pictures true only because they are useful in predicting future phenomena as completely and effortlessly as possible" (Boltzmann 1974, 261). Of course this claim might also be interpreted in plain instrumentalist (anti-realist) terms. However, further evidence that Boltzmann tended to pragmatism abounds in his *Lectures on Natural Philosophy 1903–1906*. Again and again Boltzmann employed the term *zweckmässig* (efficient) and stated that formation and appraisal of concepts, laws of thought, and scientific theories should be based on considerations of efficiency only. The reason for this is that science, like any other human activity, is a Darwinian struggle to cope with our environment: a scientific theory "should always enable us to intervene appropriately [*richtig*] in nature, so that we can attain appropriate effects" (Fasol-Boltzmann 1990, 168; cf. 85, 161, 163). In Section 3.2 it was shown that according to Boltzmann innate ideas are not infallible a priori truths but selected for their efficiency in the course of human evolution. This view leads of course straight to a pragmatist conception of truth: "Both the innate and the acquired should be tested by experience again and again; only if it proves its mettle always, if it leads to correct action always, [ . . . ] is it correct" (Fasol-Boltzmann 1990, 164). Boltzmann regards practical success as the sole criterion for being correct (*richtig*). It is telling that he always uses the term *richtig*, which means 'correct' or 'true' in a more pragmatic sense, and never the term *wahr*, which means 'true' but is more easily associated with a correspondence theory of truth.

It is crucial, in my opinion, to assess Boltzmann's inclination to a pragmatist theory of truth in the broader context of his Darwinistic worldview. Boltzmann's admiration for Darwin was unlimited and induced him to re-

gard all human cognitive faculties – and therefore science – as products of evolution. Thus he arrived at a form of naturalistic, evolutionary epistemology, in which the *Bildtheorie* is connected with a pragmatist approach to cognition: "To think means to form oneself pictures with which one can act correctly" (Fasol-Boltzmann 1990, 71). One of the implications of this all-pervading Darwinism seems to be his position with respect to scientific understanding and explanation, which is reflected in his definition of 'understanding', discussed in Section 3. In the 1902 inaugural lecture in which Boltzmann proposes this definition pragmatism appears to be a basic thread. Moreover, it contains repeated allusions to a Darwinistic worldview. The lecture begins with a description of the development of mechanics with strong emphasis on its practical significance, hinting at the survival value of mechanical tools. Boltzmann then moves on to discuss abstract analytical mechanics and arrives at his characterization of the explanatory aim of science, quoted in Section 1. This leads him to his pragmatic definition of the notion of understanding. He defends it by invoking a pragmatist theory of meaning, which is in turn illustrated with a pragmatic argument against solipsism.[15]

In conclusion, in Boltzmann's philosophy of science there is an intimate relationship between Darwinism, pragmatism, and the status and function of *Bilder*. On the one hand, Boltzmann's *Bilder* have a representative function. As representations they are 'merely' idealizations and therefore a pragmatic conception of truth is applicable, as emphasized by Blackmore (1995b, 80–1). Moreover, *Bilder* have an explanatory function. The way in which they fulfill this task, namely by being tools for understanding (and thus ultimately for survival) also has a pragmatist foundation. Obviously, these aspects are two sides of the same coin.

## 5. CONCLUSION

Boltzmann's approach to explanation and understanding was remarkably modern. Although he was a true nineteenth-century physicist who favoured mechanical explanation, Boltzmann was only too well aware of the problems and possible fate of the mechanical world-picture. For this reason, and for philosophical reasons as well (particularly his Darwinist inclinations), he refrained from endorsing an a priori justification of this form of explanation, for example by invoking a priori laws of thought. Moreover, he denied that mechanical explanation was the only possible way to achieve scientific understanding. In response to the problems of the mechanical world-picture, Boltzmann advanced the *Bildtheorie* and the associated pragmatic view of scientific understanding. The result is, as I

have attempted to argue in the present article, a philosophical approach to science that is twentieth-century-proof: Boltzmann's ideas may be employed as a basis for constructing a philosophy of science that is able to deal with the conceptual problems of contemporary science. In particular, I have indicated how his ideas may be used as a starting-point for developing a new analysis of the notion of scientific understanding. One reason for the lasting philosophical importance of Boltzmann's approach is the fact that it is not tied to a particular metaphysical worldview. As such it was able to survive the revolutionary transitions that modern physics has witnessed in the last century. It appears that philosophers who are looking for tools to tackle the epistemological problems of present-day theoretical physics could do worse than rely on Ludwig Boltzmann as a guide.

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## NOTES

[1] German original: "Es ist nun überall die Aufgabe der Wissenschaft, das Kompliziertere aus dem Einfacheren zu erklären; oder, wenn man lieber will, durch Bilder, welche dem einfacheren Erscheinungsgebiete entnommen sind, anschaulich darzustellen" (Boltzmann 1979, 195). Note concerning translations: I have used existing translations when available; in other cases the translations from German are my own.

[2] For similar formulations of the *Bildtheorie*, see Boltzmann (1974, 83, 104, 141, 225).

[3] See also Miller (1986, 91) for a discussion of this passage.

[4] Of course it can be argued that these issues are related (realism is often associated with an explanatory aim and instrumentalism with a descriptive aim). However, they do not coincide, and for our purposes it is important to distinguish clearly between them.

[5] At first glance, Boltzmann's remark might seem to support the so-called familiarity view of scientific understanding, which asserts that scientific explanations provide understanding by reducing unfamiliar phenomena to familiar ones. As many authors (e.g., Friedman 1974, 9) have argued, this view of understanding is untenable: science often explains phenomena that are directly known and quite familiar to us by means of theories and concepts of a strange and unfamiliar (sometimes even counter-intuitive) character. I will argue below that a sophisticated version of the familiarity view of scientific understanding is possible and that this is in fact what Boltzmann hinted at.

[6] See Boltzmann (1974, 149) for similar arguments in favour of mechanical explanation.

[7] See Boltzmann (1974, 105 and 195) for arguments to this effect.

[8] This book contains Boltzmann's own preparatory notes, which are often very brief, and the full text (*Mitschrift*) of the first series of his lectures, written down by an unknown member of the audience.

[9] One notices an interesting similarity between Boltzmann's pragmatic definition of understanding and the later views of Ludwig Wittgenstein on this topic. In his *Philosophical Investigations* Wittgenstein (1953, sections 151–4) discusses the problem of understanding a mathematical fomula: "A writes series of numbers down; B watches him and tries to find a law for the sequence of numbers. If he succeeds he exclaims: Now I can go on!". Perhaps B has found the formula which describes the series, say $a_n = n^2+n-1$. However, Wittgenstein adds: " 'B understands the principle of the series' surely doesn't mean simply: the formula $a_n = n^2 + n - 1$ occurs to B. For it is perfectly imaginable that the formula occurs to him and that he should nevertheless not understand". According to Wittgenstein, understanding lies in being able to use the formula, in agreement with Boltzmann's view. Although it would be too speculative to suppose a direct influence here, the similarity is remarkable (published evidence about the influence of Boltzmann on Wittgenstein is concerns only the latter's early ideas).

[10] Paul Foulkes' 1974 translation omits the phrase 'through purely mechanical pictures'. The German original is: "Wenn wir nun zum Schlusse das Resultat unserer Betrachtungen resumieren, so können wir als solches bezeichnen, dass sich eine Seite aller Vorgänge der unbelebten und belebten Natur durch rein mechanische Bilder in einer Exaktheit darstellen, wie man sich ausdrückt, begreiflich machen lässt, wie es sonst in keiner anderen Weise bisher gelungen ist, während anderseits doch alle höheren Bestrebungen und Ideale keine Embusse erleiden (Boltzmann 1979, 190).

[11] Not only Boltzmann's remarks, but also statements of Heisenberg (1983, 62) and Feynman et al. (1964, Vol. II, 2-1), suggest that this conception of intelligibility and understanding is common among scientists.

[12] The idea that scientific theories can provide understanding without having to be interpreted realistically was later defended also by Erwin Schrödinger, who was strongly influenced by Boltzmann (see De Regt 1997).

[13] Boltzmann (1964, 28–30) gives a justification of the identification of heat with molecular motion. Starting from solid bodies, and assuming that molecules are oscillating around rest positions, he then qualitatively explains phase transitions and gives a description of the molecular motion in fluid and gaseous states. Feynman et al. (1963, Vol. I, 1–2ff.) present a similar argument.

[14] Blackmore (1995b, 81) adds, however, that Boltzmann's attachment to pragmatism "was not complete since he could not go all the way maintaining with James [ ... ] that what didn't work was 'meaningless' ".

[15] Boltzmann (1974, 150) claims that he had proven solipsism false when he discovered that "it led to my failing to take the right practical action and caused me great damage".

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